INTERNATIONAL ECONOMIC JOURNAL
Volume 15, Number 1, Spring 2001

CURRENCY HEDGING FOR EXPORT-FLEXIBLE FIRMS*

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This paper examines the production and hedging decisions of a competitive exporting firm under exchange rate uncertainty. The firm possesses export flexibility in that it can distribute its output to either the domestic market or a foreign market, after observing the true realization of the exchange rate. It is shown that the separation theorem does not hold under export flexibility, i.e., the firm's optimal output depends on the firm's preference and on the underlying exchange rate uncertainty. Furthermore, the export-flexible firm underhedges its exchange rate risk exposure in a currency forward market wherein the forward exchange rate contains a non-positive risk premium. [D21, F31]

1. INTRODUCTION

Since the collapse of the Bretton Woods Agreement in 1973, exchange rates have become substantially volatile (see Meese, 1990). In response to increased exchange rate fluctuations, international firms have been devising various hedging strategies to reduce their foreign exchange risk exposures. On the one hand, they can adopt a real hedge by following a flexible input/output policy which allows them to alter their operations according to realized exchange rates. On the other hand, they can opt for a financial hedge by trading currency derivatives such as futures and options. To understand the hedging behavior of international firms, the interaction between real and financial hedging deserves closer scrutiny.

The extant literature on the competitive exporting firm under exchange rate uncertainty typically assumes that the firm makes its production, export, and hedging decisions simultaneously (see, e.g., Katz and Paroush, 1979; Benninga, Eldor, and Zilcha, 1985; Kawai and Zilcha, 1986; Adam-Müller, 1997; and Broll, Wong, and Zilcha, 1999). Two notable results emanate. First, the "separation theorem" states that the production decision of the firm is affected neither by the risk attitude of the firm nor by the incidence of exchange rate uncertainty should the firm have access to a currency forward market.[1] Second, the "full-hedging

*I would like to thank Axel Adam-Müller, Udo Broll, and three anonymous referees for helpful comments and suggestions. Any remaining errors are of course mine.

[1]The separation theorem in the hedging literature is somewhat related to Fisher's (1930) separation theorem which states that, given perfect capital markets, individuals choose among assets by comparing present values using a market rate of interest, and their choices are not

theorem" states that the firm should completely eliminate its exchange rate risk exposure by holding a full hedge if the currency forward market is unbiased.

The purpose of this paper is to re-examine these two well-established results in the context of an export-flexible firm under exchange rate uncertainty *à la* Broll (1999). The firm is export flexible in the following sense. While production takes place prior to the resolution of exchange rate uncertainty, the firm makes its export decision (i.e., sales allocation between the domestic market and a foreign market) only after observing the realized exchange rate.[2] The firm's optimal export decision is state contingent: exports the entire output to the foreign market when the realized exchange rate is sufficiently favorable such that the foreign price (measured in units of the domestic currency) exceeds the domestic price; otherwise, sells exclusively in the domestic market. Export as such can be viewed as a real option with a "strike price" equal to the domestic price. Multinationals, because of their worldwide distribution facilities, fit particularly well the context of export-flexible firms (see Caves, 1996; Broll, 1999; and Broll and Eckwert, 1999).

Under export flexibility, the firm receives downside protection from selling in the domestic market at the domestic price, and concomitantly enjoys all the upside potentials from exporting to the foreign country at favorable realizations of the exchange rate. This call option like feature of the firm's state-contingent export rule essentially convexifies the firm's linear profit function, thereby making the firm more aggressive in production. We show that the separation theorem fails to hold. The non-linearity of the firm's profit function cannot be perfectly offset by trading linear currency forward contracts. The firm's optimal output as such depends on the firm's preference as well as on the underlying exchange rate uncertainty. Furthermore, we show that the firm's optimal forward position is always an under hedge as long as there is a non-positive risk premium embedded in the forward exchange rate. Thus, the full-hedging theorem is also invalidated under export flexibility.

The rest of the paper is organized as follows. The next section delineates the model of an export-flexible firm under exchange rate uncertainty. Section 3 characterizes the firm's optimal production decision when a currency forward market is available. Section 4 derives the firm's optimal forward position. The final section concludes.

2. THE MODEL

Consider a risk-averse competitive firm which produces a single output, Q, according to an increasing cost function, $C(Q)$. The firm can sell its output either in the domestic market at a per-unit price, P_d, denominated in the domestic

influenced by their preferences about the timing of consumption.

[2]Ben-Zvi and Helpman (1992) argue that international transactions are better described by such a sequence of moves. This is supported by the empirical evidence in Magee (1974).

currency, or in a foreign market at a per-unit price, P_f, denominated in the foreign currency. When the firm makes its production decision, it does not know the exchange rate of the domestic currency against the foreign currency, e. The firm treats e as a random variable distributed over support $[\underline{e}, ç]$ according to a cumulative distribution function, $F(e)$, where $0 \leq \underline{e} < ç \leq \infty$. The two per-unit prices, P_d and P_f, are fixed and known to the firm. Due to the segmentation of the domestic and foreign markets, arbitrage transactions are either impossible or unprofitable, thereby hindering the law of one price.

Prior to making its export decision, the firm observes the true realization of e. Given this observed e, the firm's optimal export decision is trivial: exports the entire output to the foreign market if $eP_f > P_d$; or sells exclusively in the domestic market if $eP_f < P_d$. The firm is indifferent between selling to either market if, and only if, $eP_f = P_d$, which occurs with probability zero. The sales revenue of the firm as such can be compactly written as max $[eP_f, P_d]Q$, revealing a convex pattern in e. In other words, the flexibility in making export decisions after observing the true realization of e provides the firm a valuable real (call) option which is exercised if e is sufficiently high (i.e., $e > P_d / P_f$).

To hedge against its exchange rate risk exposure, the firm can trade infinitely divisible forward contracts in a currency forward market. Figure 1 depicts how the sequence of events unfolds in the model. At time 0 (see figure 1), the firm sells (purchases if negative) H units of the foreign currency at a forward rate, e_0, expressed in units of the domestic currency per unit of the foreign currency at time 2. Since the exchange rate uncertainty is completely resolved at time 1, the firm, being risk averse, will not opt for a new forward position at time 2, unless there is a sufficiently high risk premium embedded in the time 2 forward exchange rate, which is unlikely.

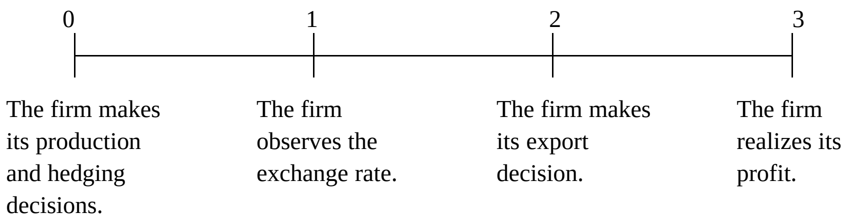

Figure 1. Time Line

Anticipating the *ex post* contingent export decision described above, the firm chooses an output level, Q, and a forward position, H, at time 0 so as to maximize the expected utility of its profit, EU:

$$\max_{Q, H} \ EU = \int_{P_d/P_f}^{\bar{e}} U\left[\Pi_f(e)\right] \ dF(e) + \int_{\underline{e}}^{P_d/P_f} U\left[\Pi_d(e)\right] \ dF(e),$$

$$\text{s.t.} \quad \Pi_f(e) = eP_f Q + (e_0 - e)H - C(Q),$$
$$\Pi_d(e) = P_d Q + (e_0 - e)H - C(Q),$$

where U is a von Neumann-Morgenstern utility function with $U' > 0$ and $U'' < 0$, indicating the presence of risk aversion. The first-order conditions for an optimum are given by

$$\int_{P_d/P_f}^{\bar{e}} U'\left[\Pi_f^*(e)\right]\left[eP_f - C'(Q^*)\right] dF(e) + \int_{\underline{e}}^{P_d/P_f} U'\left[\Pi_d^*(e)\right]\left[P_d - C'(Q^*)\right] dF(e) = 0, \tag{1}$$

$$\int_{P_d/P_f}^{\bar{e}} U'\left[\Pi_f^*(e)\right](e_0 - e)\, dF(e) + \int_{\underline{e}}^{P_d/P_f} U'\left[\Pi_d^*(e)\right](e_0 - e)\, dF(e) = 0, \tag{2}$$

where an asterisk (*) indicates an optimal level. The second-order conditions for a maximum are assumed to be satisfied.[3]

3. OPTIMAL PRODUCTION DECISIONS

To examine the optimal output of the firm, we multiply P_f to equation (2) and add the resulting equation to equation (1) to yield

$$e_0 P_f - C'(Q^*) = \frac{\int_{\underline{e}}^{P_d/P_f} U'[\Pi_d^*(e)](eP_f - P_d)\, dF(e)}{\int_{P_d/P_f}^{\bar{e}} U'[\Pi_f^*(e)]\, dF(e) + \int_{\underline{e}}^{P_d/P_f} U'[\Pi_d^*(e)]\, dF(e)}. \tag{3}$$

[3]Given risk aversion, a sufficient (but not necessary) condition to ensure the second-order conditions to hold is that $C'' \geq 0$.

If $P_d \leq \underline{e}P_f$, the numerator on the right-hand side of equation (3) vanishes so that $e_0 P_f - C'(Q^*) = 0$. However, if $P_d > \underline{e}P_f$, the numerator on the right-hand side of equation (3) is strictly negative so that $C'(Q^*) > e_0 P_f$. Thus, we establish the following proposition.

PROPOSITION 1. *When the currency forward market is available, the export-flexible firm's optimal output, Q^*, is implicitly defined by $C'(Q^*) = e_0 P_f$, if, and only if, selling in the domestic market is completely dominated by selling in the foreign market* (*i.e.*, P_d £ $\underline{e}P_f$).

If the domestic price is so low that it is never optimal to sell in the domestic market (i.e., P_d £ $\underline{e}P_f$), then, using the jargon of derivatives traders, the real option held by the firm is said to be deeply "out of the money" and thus has zero option value of waiting. The export-flexible firm is observationally equivalent to an otherwise identical firm which is obliged to export its entire output to the foreign country. The well-known separation theorem applies in that an export-inflexible firm's optimal output depends neither on the firm's utility function nor on the exchange rate uncertainty in the presence of a currency forward market. Since the requirement that P_d £ $\underline{e}P_f$ is unduly strong, the validity of the separation theorem should be an exception rather than a norm under export flexibility.

Now consider the hypothetical case where the firm is restricted to sell only in the domestic market. In this autarky case, the firm's optimal output, Q^0, is governed by the point at which the marginal cost of production equals the domestic price, i.e., $P_d = C'(Q^0)$. Evaluating the left-hand side of equation (1) at $Q = Q^0$ and $H = H^*$ and using the fact that $P_d = C'(Q^0)$ yields

$$\int_{P_d/P_f}^{\bar{e}} U'\left[eP_f Q^0 + (e_0 - e)H^* - C(Q^0)\right](eP_f - P_d)\, dF(e) > 0,$$

where the inequality follows from the fact that $eP_f - P_d > 0$ for all $e \in (P_d / P_f, \ \bar{e}\,]$. As long as selling in the foreign market is not completely dominated by selling in the domestic market (i.e., $\bar{e}P_f > P_d$), equation (1) and the second-order conditions imply that $Q^* > Q^0$, thereby invoking the following proposition.

PROPOSITION 2. *If selling in the foreign market is not completely dominated by selling in the domestic market* (*i.e.*, $\bar{e}P_f > P_d$), *opening up the foreign market to the export-flexible firm induces the firm to produce more.*

The intuition of Proposition 2 is as follows. Under export flexibility, the firm would export its entire output to the foreign country if the realization of the exchange rate is sufficiently favorable (i.e., $e > P_d / P_f$); otherwise, it sells exclusively in the domestic market. The firm as such receives downside protection from selling in the domestic market at the certain domestic price, P_d, and concomitantly enjoys all the upside potentials from exporting to the foreign country at favorable realizations of the exchange rate. Figure 2 depicts the firm's profit as a function of e when $H = 0$. Opening up the foreign market to the firm essentially convexifies the firm's linear profit function represented by the dashed line. It is this convexity which induces the firm to produce more when the foreign market is made accessible to the firm.

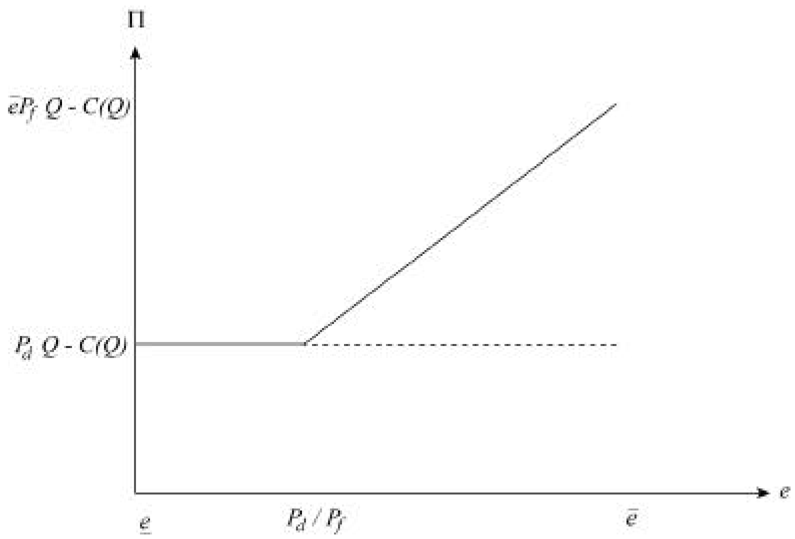

Figure 2. Profit Function for Zero Forward Position

4. OPTIMAL HEDGING DECISIONS

In this section, we want to characterize the firm's optimal forward position, H^*. To this end, we differentiate EU with respect to H and evaluate the resulting derivative at $Q = Q^*$ and $H = P_f Q^*$ to yield

$$\frac{\partial EU}{\partial H} = \int_{P_d/P_f}^{\bar{e}} U'[\Pi_f^*(e_0)]\,(e_0 - e)dF(e) + \int_{\underline{e}}^{P_d/P_f} U'[\hat{\Pi}_d^*(e)](e_0 - e)\, dF(e), \tag{4}$$

where $\prod_f^*(e_0) = e_0 P_f Q^* - C(Q^*)$ and $\hat{\prod}_d^*(e) = P_d Q^* + (e_0 - e) P_f Q^* - C(Q^*)$. If the sign of the right-hand side of equation (4) is negative (positive), equation (2) and the second-order conditions would imply that $H^* < (>) P_f Q^*$.

PROPOSITION 3. *If a non-positive risk premium is embedded in the forward exchange rate* (*i.e.*, $E(e) \geq e_0$), *the export-flexible firm's optimal forward position,* H^*, *is an under hedge* (*i.e.*, $H^* < P_f Q^*$).

The proof of Proposition 3 is relegated to the appendix. To see the intuition of Proposition 3, we refer to Figure 3.

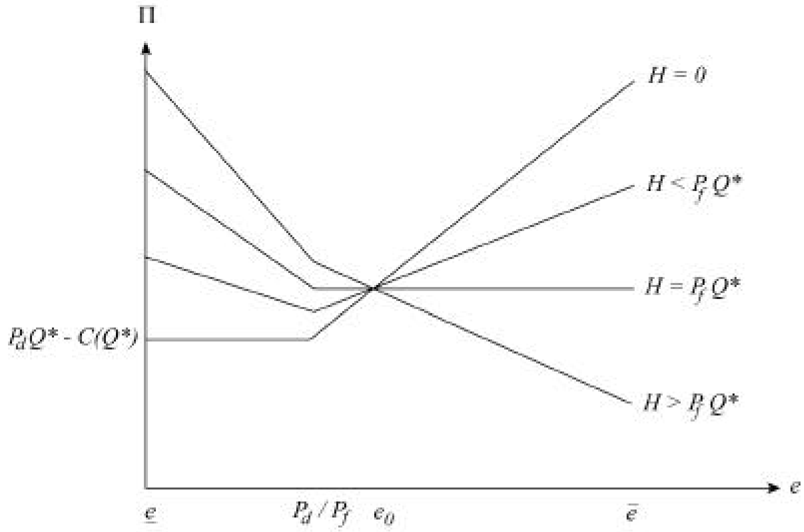

Figure 3. Profit Functions for Different Forward Positions

Figure 3 depicts the firm's profit as a function of e for four different forward positions under the assumption that $e_0 > P_d / P_f$.[4] It is evident from Figure 3 that an over hedge (i.e., $H > P_f Q^*$) induces a higher variability in the firm's profit than a full hedge (i.e., $H = P_f Q^*$). Given risk aversion, the former is dominated by the latter. An under hedge (i.e., $H < P_f Q^*$), on the other hand, induces a higher variability in the firm's profit than the full hedge only when $e > P_d / P_f$. For $e <$

[4]The case where $e_0 \leq P_d / P_f$ can be analyzed analogously.

P_d / P_f, the under hedge makes the firm's profit less variable as compared to the full hedge. Since the risk-averse firm pays more attention to the downside than to the upside, it would optimally underhedge its exchange rate risk exposure. Indeed, it is quite possible that the export-flexible firm would refrain from doing any hedging at all.

5. CONCLUSIONS

This paper has re-examined the separation theorem and the full-hedging theorem for an export-flexible firm under exchange rate uncertainty *à la* Broll (1999). Export flexibility has been shown to introduce a non-linearity in the firm's profit function, as contrasted to the linear profit function normally assumed in the extant literature. It is this non-linearity which invalidates the separation and full-hedging theorems for the export-flexible firm.[5]

In principle, currency options, because of their asymmetric payoff profiles, should be a better hedging instrument for export-flexible firms than currency forward contracts. Indeed, as shown by Broll and Wahl (1997), export-flexible firms can completely eliminate their exchange rate risk exposures by writing call options. Empirically, Ahmadi, Sharp, and Walther (1986) document that currency futures are better than currency options when hedging downside risk of the British pound, the Deutsche mark, and the Japanese yen (see also Chang and Shanker, 1986). Benet and Luft (1995) show further that the hedging effectiveness of futures contracts over options improves once transaction costs and margin requirements are taken into account. In light of these empirical observations, hedging with currency forward contracts remains important to export-flexible firms and its economic implications would provide useful guidelines for these firms when facing exchange rate uncertainty.

APPENDIX

Consider first the case where $P_d \leq e_0 P_f$. In this case, we have $e_0 \geq e$ for all $e \in [\underline{e}, P_d/P_f]$. Rearranging terms of equation (4) yields

$$\frac{\partial EU}{\partial H} = U'\left[\Pi_f^*(e_0)\right]\left[e_0 - E(e)\right] + \int_{\underline{e}}^{P_d/P_f} \left\{U'\left[\hat{\Pi}_d^*(e)\right] - U'\left[\Pi_f^*(e_0)\right]\right\}(e_0 - e)\, dF(e). \quad \text{(A1)}$$

[5] Adam-Müller and Wong (2000) show that introducing restrictions in export flexibility in that the firm needs to maintain certain minimum levels of domestic sales and exports affects none of the qualitative results in this paper.

Since $U'' < 0$ and $\hat{\Pi}_d^*(e) - \Pi_f^*(e_0) = (P_d - eP_f)Q^* \geq 0$ for all $e \in [\underline{e}, P_d/P_f]$, we have $U'[\hat{\Pi}_d^*(e)] \leq U'[\Pi_f^*(e_0)]$. Thus, the second term on the right-hand side of equation (A1) is unambiguously negative. Inspection of equation (A1) reveals that ¶*EU*/¶*H* < *0* whenever $e_0 \leq E(e)$.

Now, consider the case where $P_d > e_0P_f$. Rewrite equation (4) as

$$\frac{\partial EU}{\partial H} = U'\left[\hat{\Pi}_d^*(e_0)\right]\left[e_0 - E(e)\right] + \left\{U'\left[\Pi_f^*(e)\right] - U'\left[\hat{\Pi}_d^*(e_0)\right]\right\}\int_{P_d/P_f}^{\bar{e}}(e_0 - e)\, dF(e)$$

$$+ \int_{\underline{e}}^{P_d/P_f}\left\{U'\left[\hat{\Pi}_d^*(e)\right] - U'\left[\hat{\Pi}_d^*(e_0)\right]\right\}(e_0 - e)\, dF(e). \qquad \text{(A2)}$$

Since $U'' < 0$ and $\partial\hat{\Pi}_d^*(e)/\partial e = -P_fQ^* < 0$, we have

$$U'\left[\hat{\Pi}_d^*(e)\right] \leq (\geq)\ U'\left[\hat{\Pi}_d^*(e_0)\right] \quad \text{for } e \leq (\geq)\, e_0.$$

Thus, the third term on the right-hand side of equation (A2) is unambiguously negative. Given that $P_d > e_0P_f$, we have $\hat{\Pi}_d^*(e_0) - \Pi_f^*(e_0) = (P_d - e_0P_f)Q^* > 0$. It follows from risk aversion that $U'[\Pi_f^*(e_0)] > U'[\hat{\Pi}_d^*(e_0)]$. For $e_0 \leq E(e)$, we have $\int_{P_d/P_f}^{\bar{e}}(e_0 - e)\, dF(e) < 0$. Thus, the second term on the right-hand side of equation (A2) is negative if e_0 £ *E*(*e*). Inspection of equation (A2) reveals that ¶*EU* / ¶*H* < *0* whenever e_0 £ *E*(*e*).

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