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OPTIMAL CURRENCY TARGET ZONES: HOW WIDE SHOULD EXCHANGE RATE BANDS BE?

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This paper presents a model of an optimal currency band in which a central bank with an infinite time horizon faces a trade-off between interest rate deviation costs and exchange rate deviation costs. The bank chooses optimal intervention points in order to minimize the value of the loss function. The paper uses the Bellman inequalities for instantaneous control of the regulated Brownian motion to derive an optimal currency band and optimal intervention policy characterized by two barriers. This model derives some interesting results. First, the width of currency band depends positively on the uncertainty of the shock, the degree of speculative pressure, and central bank's concern about the domestic money market versus the foreign exchange market. Second, the central bank finds it optimal not to intervene when the fundamental rate is inside a certain band, whereas once it lies outside the band, the optimal policy is to move it to the nearest boundary. [F31]

1. INTRODUCTION

Recently, a number of research efforts have focused on the issue of how foreign exchange market intervention operates under a target zone exchange rate regime. Krugman (1987) emphasized the role of official intervention at the margin of a currency band. This type of intervention is referred to as “marginal” intervention as opposed to “intramarginal” intervention which represents intervention within the upper margin of the band.

A credible commitment to intervene at the margin is enough to generate an S-shaped relationship between the exchange rate and the fundamental; the exchange rate moves with the level of the fundamental within the band, and at the edge of the band it tends to move back towards the middle. The implication is that active intervention is unnecessary inside the band. Froot and Obstfeld (1989) argued that marginal intervention to defend a currency band should take place through continuous infinitesimal adjustments. Bertola and Caballero (1989) examined the implications of infinitesimal intervention for exchange rates in a currency band with realignment options. Svensson (1989) studied the behavior of interest rates

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in an exchange-rate target-zone regime, assuming that intervention would be infinitesimal and take place only at the margin of a currency band.

Flood and Garber (1989) addressed another type of intervention policy: *discrete* or finite intervention. Observing the actual intervention conducted by the German Bundesbank that employed large scale reserve operations, they argued that the intervention took the form of a *discrete* change in foreign reserves. Pesenti (1990) extended the Krugman model to account for the role of intramarginal intervention. Dominguez and Kenen (1992) empirically showed that since the introduction of the Basle-Nyborg Agreement of 1987 in the European Monetary System (EMS), intramarginal intervention has been actively employed by the member central banks.

All the above studies provide a positive framework for analyzing the nature of intervention in the EMS. However, since they implicitly assume that a currency band is the ideal monetary structure, they fail to predict what type of intervention will be required to maintain the currency band, in terms of its timing and magnitude. In other words, they leave unanswered the issue of *when* and *how* intervention in the foreign exchange market should take place.

This paper intends to bridge the gap by explicitly introducing an optimizing framework to determine the ideal size of a currency band. Interestingly, Miller and Zhang (1994) recently developed a model of optimal target zones in which the central bank tries to minimize the exchange rate deviation costs, assuming that an exchange rate solution inside any sized band follows an *S*-shaped pattern. This paper extends Miller and Zhang's model by assuming that the central bank is concerned with the interest rate deviation as well as the exchange rate deviation. This addition to the central bank's objective makes this model more realistic and hence more relevant than that of Miller and Zhang, since the central bank under the target zone exchange rate regime faces interest rate volatility. Volatility of interest rates is a natural side effect of intervention; stabilization of the exchange rate is obtained at the expense interest rate stability (Flood and Garber 1989 and Svensson 1990).

In addition, our model uses the Bellman inequality conditions to explicitly demonstrate that the value function derived by Miller and Zhang satisfies the optimality conditions.

This paper presents the optimal intervention strategies of a central bank with an infinite horizon facing proportional intervention costs. Since the intervention to eliminate the currency fluctuations produces some interest rate deviation, the central bank faces a trade-off between these two types of costs. Given this situation, the central bank seeks to find the optimal intervention policy which minimizes the value of the loss function.

The optimal policy involves two boundaries for the fundamental characterized by two reflecting barriers. When the fundamental lies inside a band composed of a lower and upper boundary, the central bank finds it optimal not to intervene. When the fundamental hits one of these bounds, the central bank intervenes in order to keep it within in the band.

Methodologically, this model uses the optimality conditions of the Bellman equation for the reflected Brownian motion process (Dixit 1990, Fleming et al. 1993, and Harrison and Taksar 1983).

The results of this model are as follows: compared to the size of the currency band proposed by Miller and Zhang (1994), the optimal band derived here is narrower. Since the central bank's intervention to stabilize currency fluctuations destabilizes short-term interest rates, the central bank's concern about interest rate deviations makes the marginal deviation cost higher than that derived by Miller and Zhang. In order to exactly match the marginal deviation cost with the intervention cost, the central bank finds it optimal to intervene earlier than in case of the model by Miller and Zhang.

The second key result of the analysis will show that the optimal size of the intervention band depends positively on all of the following: the volatility of the fundamental; the extent of the central bank's intervention cost characterized by foreign exchange transaction costs; the height of the time-discount rate; the degree of priority reflecting the central bank's concern about the domestic money market; and the strength of speculative pressure.

The analysis shows that the optimum intervention policy leads the central bank to not intervene when the exchange rate lies inside the band. Once it hits the edge, the optimal policy is to keep the exchange rate within the band. In this case, the optimal intervention is of the infinitesimal type suggested by Krugman (1987).

This paper is structured as follows. Section 2 will review some stylized facts about EMS intervention. Section 3 will lay out a model of a small open economy in which the central bank is concerned with the stability of the domestic financial markets as well as that of the foreign exchange market. Section 4 will examine the behavior of a free-floating exchange rate regime as a simple example of the model. Section 5 will model an optimal control policy in an exchange rate target-zone economy and section 6 will compare our model with that of Miller and Zhangs. Section 7 will use a numerical simulation to characterize the response of optimal band to changes in the underlying parameters and Section 8 will summarize the paper and provide some conclusions.

2. THE EMS INTERVENTION

This section provides a brief review of how the EMS exchange rate and intervention mechanism worked as of 1992.

A. The EMS Agreements

By joining the Exchange Rate Mechanism (ERM) of the EMS, each currency was assigned a central exchange rate vis-a-vis the European Currency Unit (ECU), a basket of member currencies.[1] The ratio of any two ECU central rates established a bilateral central exchange rate. Any bilateral central exchange rate was fixed but it could be altered by common consent rather than by a unilateral decision.

The member currencies were allowed to fluctuate within limited bands around their central rates. All currencies, except the peseta and sterling, were allowed to move plus or minus 2.25 percent from their central rates. (The fluctuation bands for the peseta and sterling were 6 percent.)

Each member central bank participating in the ERM had an obligation to intervene in the foreign exchange market when two currencies reached an upper or lower limit of the band; both monetary authorities involved had to make the purchases or sales of international reserves expended in the foreign exchange market to prevent the exchange rate from moving beyond the fluctuation band.

Suppose that a strong currency, for example the Deutsche mark (DM), reaches the lower margin of the band against a weak currency for example, the French franc. The agreement of the EMS dictated that the Bundesbank must swap DM for ECUs through the very-short-term financing facility (VSTFF) and the Bank of France must swap francs for ECUs. Then the borrowed ECUs are exchanged for DMs by the Bank of France and used for exchange market intervention. Borrowings from the VSTFF would be repaid within three and a half months, with the possibility of a further three month extension.

Alternatively, the Bank of France could borrow DMs on a bilateral basis from other banks, mainly the Bundesbank, for marginal intervention. In contrast to the use of ECU credit through the VSTFF, bilateral loans would not be denominated in ECUs and would not be subject to a rigid repayment schedule.

However, even if the EMS agreement of March 1979 envisaged marginal intervention as a main intervention instrument to maintain stable exchange rates in the EMS, the EMS agreement allowed each central bank to intervene even before the exchange rate hit the fluctuation band. This (voluntary) intramarginal intervention was subject to the approval of the central bank whose currency was being used. In addition, intramarginal intervention had to be financed either out of reserve holdings or by borrowing from the bilateral credits of other central banks. After the Basle/Nyborg agreement of September 1987, central banks had limited access to the VSTFF for intramargial intervention. The next subsection will examine how actual EMS intervention took place.

B. Evidence of EMS Intervention

Table 1 presents the evidence of the increasing importance of intramarginal intervention over time. The data on the EMS intervention represents DM interventions undertaken by ERM member central banks including the Bundesbank.

[1] In the beginning of the EMS, the central exchange rate reflected the rate that had been prevailing in the market.

Plus signs indicate DM sales whereas minus signs indicate DM purchases. For example, -2.7 for total interventions in 1979 indicates that in 1979 the member countries bought 2.7 billion DM for EMS intramarginal intervention, and that the Bundesbank purchased 2.4 (billion DM) of DM reserves for intramarginal intervention.

Table 1. DM in the EMS (in billions of DM)

		Obligatory	Intramarginal	Total	In Fed. Rep. Of Germany
1979	Purchases	-	-2.7	-2.7	-2.7
	Sales	+3.6	+8.1	+11.7	+11.7
	Balances	+3.6	+5.4	+9.0	+9.2
1980	Purchases	-5.9	-5.9	-11.8	-11.4
	Sales	-	+1.0	+1.0	+0.6
	Balances	-5.9	-4.9	-10.8	-10.5
1981	Purchases	-2.3	-8.1	-10.4	-11.6
	Sales	+17.3	+12.8	+30.1	+25.3
	Balances	+15.0	+4.7	+19.7	+13.7
1982	Purchases	-	-9.4	-9.4	-2.5
	Sales	+3.0	+12.8	+15.8	+6.1
	Balances	+3.0	+3.4	+6.4	+3.6
1983	Purchases	-16.7	-19.1	-35.8	-20.4
	Sales	+8.3	+12.9	+21.2	+12.6
	Balances	-8.4	-6.2	-14.5	-7.8
1984	Purchases	-	-28.9	-28.9	-3.0
	Sales	+4.7	+7.6	+12.3	+4.4
	Balances	+4.7	-21.4	-16.6	+1.4
1985	Purchases	-	-29.1	-29.1	-0.2
	Sales	+0.4	+30.8	+31.1	-
	Balances	+0.4	+1.6	+2.0	-0.2
1986	Purchases	-19.0	-33.6	-52.6	-12.2
	Sales	+4.1	+74.0	+78.1	+3.8
	Balances	-14.8	+40.4	+25.5	-8.4
1987	Purchases	-	-47.8	-47.8	-7.3
	Sales	+15.0	+61.7	+76.8	+25.4
	Balances	+15.0	+13.9	+28.9	+18.1
1988	Purchases	-	-26.8	-26.8	-6.1
	Sales	-	+16.3	+16.3	-
	Balances	-	-10.5	-10.5	-6.1
1989	Purchases	-	-20.4	-20.4	-3.0
	Sales	+5.0	+8.6	+13.6	+3.0
	Balances	+5.0	-11.8	-6.8	0.0

Note: The European Monetary System: Developments and Perspectives, IMF Occasional Paper No.73.

In addition, Table 1 shows that no marginal interventions were undertaken in 1979. Since the beginning of the EMS period, the central banks have preferred intramarginal intervention to marginal intervention. The table shows the increasing use of intramarginal intervention over marginal intervention, particularly after the Basle/Nyborg agreement of 1987. In 1988, no marginal intervention took place, and in 1989 all the DM purchase activities were undertaken in the form of intramarginal intervention. In fact, in 1989, the value of all the reserves expended by intramarginal intervention reached more than 60 billion DM.

Table 2 presents the evidence of EMS foreign exchange intervention. It breaks down EMS intervention by type of intervention in the subperiods January 1983-March 1985 and April 1985-April 1986.[2] The table shows cumulative intervention figures as a percentage of total interventions carried out by all countries in each subperiod. The negative signs in the table indicate a sale of foreign reserves by the central bank. For instance the figure for the Bundesbank. –0.0093, indicates that it undertook 0.93 percent of all the marginal interventions in the form of foreign reserve sales in the first subperiod. Italy did not intervene at the margin in any subperiod while France did so consistently; the Bank of France conducted 31.3 and 86.1 percent of all marginal interventions in the first and second subperiods, respectively.

Table 2. Breakdown of the EMS Intervention According to the Type of Intervention

	Jan. 1983 – March 1985	Apr. 1985 – Apr. 1986
Marginal Intervention		
Germany	(-) 0.0093	(-) 0.031
Netherlands	(-)0.039	(-)0.108
France	0.313	0.861
Italy	0.000	0.000
Belgium	(-)0.554	0.000
Intramarginal Intervention		
Germany	0.000	0.000
Netherlands	0.006	0.051
France	0.637	(-)0.647
Italy	0.073	(-)0.079
Belgium	0.283	(-)0.224

Note: Compiled from Giavazzi et al (1989: 65).

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[2] This information on EMS interventions is incomplete in the sense that it does not cover a whole EMS period. In addition, note that it is not possible to gather complete data on intervention due to confidentiality.

The Bundesbank did not undertake intramarginal intervention in either subperiod. Conversely, the Bank of Italy conducted about 7 percent of all intramarginal intervention in each period and the Bank of France conducted the most active intramarginal intervention: more than 60 percent of all intramarginal intervention in each period.

From an examination of the data, it can be inferred that most of the intramarginal intervention was conducted by central banks other than the Bundesbank. However, Germany did conduct interventions whenever its exchange rate hit the margin.

3. THE MODEL

This section will provide the basic framework of the model developed in this paper for studying interventions in an exchange rate target zone regime. The building blocks of the model are the exchange rate determination model and the microstructure of two financial markets.

A. The Exchange Rate Dynamics

First, consider a small open economy characterized by the (loglinear) continuous-time version of a monetary approach to exchange rates:[3]

$$e(t) = f(t) + d\,\frac{E[de(t) \mid \Phi(t)]}{dt} \tag{1}$$

with

$$f(t) = m(t) + v(t) \tag{2}$$

where $e(t)$ is the log of the price of foreign exchange in terms of domestic prices; $E[de(t) \mid \Phi(t)]/dt$ is the conditional expected depreciation rate; and $\Phi(t)$ is the information set that agents in the foreign exchange market are assumed to use to make their decisions. The log of the fundamental, $f(t)$ is the sum of the log of the money supply, $m(t)$ and the log of the money demand shock, $v(t)$. The expression

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[3] Equation (1) is derived in Appendix.

$m(t)$ is a policy variable and is used to define an exchange rate regime. Under a free-floating exchange rate regime, no intervention occurs, so $m(t)$ is constant. In a target zone economy, $m(t)$ is changed to influence the exchange rate. The parameter d represents the sensitivity of the demand for money with respect to the interest rate. It may be interpreted as the degree of speculative pressure on the exchange rate.

An exogeneous source of shocks to the economy, $v(t)$, is the forcing variable which drives the dynamics of the economy. Let us assume that $v(t)$ follows a Brownian motion process with no drift term and instantaneous standard deviation s:

$$dv(t) = s\, dw(t) \quad (3)$$

where

$$v(0) = v_0 \quad constant$$

$dw(t)$ is the increment of a standard Wiener process with $E(dw(t)) = 0$, $Var(dw(t)) = t$.

B. The Loss Function of the Central Bank with the Linear Transaction Cost

In this economy, the fundamental $f(t)$ tends to deviate from the initial value over time. These fluctuations expose dealers in the foreign exchange market to exchange rate risk driven by the uncertainty of the fundamental. The central bank may dislike disturbances of the fundamental becsuse they create volatility in the exchange rate. This concern motivates the central bank to intervene to stabilize currency fluctuations. Intervention takes the form of sales and purchases of foreign reserves held by the central bank.

Intervention adversely affects the domestic money market, however, by creating volatility in the interest rates. First, suppose that to avoid further depreciation of the domestic currency, the central bank conducts sales of foreign exchange. This implies squeezing liquidity in the domestic money market. In this case, money market players face the increasing risk of being unable to acquire funds. To compensate for the higher risk, they have to widen the bid-ask spread. This scenario would probably also cause a decrease in the volume of transactions.

Second, intervention-induced interest rate volatility may have adverse effects on government finances. If a government issues its securities with a short-term maturity and floating rates, variability of the interest rate puts the government in a difficlult position to refinance the securities, especially if using a roll-over financing method. The central bank should take into account all undesirable side effects in deciding whether or not it will intervene.

The central bank is responsible for the stability of both the money market and the currency market under a target zone exchange rate regime. To capture the trade-off that the central bank faces, this paper assumes that the loss function of the central bank is composed of two forms of costs: the exchange rate deviation cost and the interest rate deviation cost. Specifically, exchange rate deviation costs are expressed as the square of the difference between the market exchange rate and the target rate, with the target set at zero. Interest rate deviation costs are also expressed in this fashion. In a target zone regime, interest rate deviation cost is produced by foreign exchange market intervention to stabilize exchange rate fluctuations.

The Specification of the Value Function:

Let us denote the loss function of the central bank as:

$$\begin{aligned} J(f:U,L) = E_0 \int_0^{\infty} exp^{-rt}\,[(1-a)(e(t)-\bar{e})^2 + a(i(t)-\bar{i})^2]dt \\ + b\int_0^{\infty} exp^{-rt}\,(U+L)dr \end{aligned} \tag{4}$$

where

$e(t)$: level of the exchange rate at time t.
$\bar{e}$: central bank's target exchange rate which is assumed to be fixed at 0.
$(e(t))^2$: exchange rate deviation cost.
r: central bank's discount rate.
$i(t)$: domestic instantaneous interest rate prevailing at time t.
$\bar{i}$: central bank's target interest rate.
$(i(t)-\bar{i})^2$: interest rate deviation.
a : degree of priority which the central bank places on interest rate fluctuation relative to exchange rate deviation.
b : unit transaction cost associated with the sales(purchases) of foreign reserves.[4]
U: (instantaneous) growth of the cumulative gross sales of foreign exchange reserves
L: (instantaneous) growth of the cumulative gross purchases of foreign exchange reserves.

[4] b may be interpreted as the bid (ask) price for foreign exchange transactions.

Equation (4) represents the expected discounted value of the sum of future deviation costs and control costs incurred when the fundamental, starting from *f*, fluctuates in a pattern conforming to a regulated Brownian motion. The deviation costs are composed of the exchange rate deviation and the interest rate deviation. The objective of the central bank is to minimize the loss function. The value function, *V*(*f: U, L*) that represents the minimized loss function of the central bank is defined as:

$$V(f:U,L)=\min_{U.L} J(f:U,L) \tag{5}$$

Equation (5) implies that the central bank finds a policy that minimizes the expected discounted sum of these costs over an infinite planning horizon for every starting state of variable *f*.

The Specification of the Dynamics of Intervention Strategies:

The Brownian motion specification of the fundamental is assumed to be regulated by the following reflecting intervention strategies:

$$\begin{aligned} df(t) &= s\ \ dw(t)\ \ if\ \ \ f\in[f_l, f_u] \\ &= s\ \ dw(t)-dU\ \ if\ \ \ f\geq f_u \\ &= s\ \ dw(t)+dU\ \ if\ \ \ f\leq f_l \end{aligned} \tag{6}$$

where f_u and f_l represent the upper (lower) bound of the fundamental, respectively.

This specfication of the dynamics of the fundmental deserves some elaboration. First of all, a policy is defined as pair of f_u and f_l and the associated *U* and *L* processes. Second, this economy has an optimal control policy characterized by two symmetrical barriers with a upper and lower barrier (f_u f_l) with f_l = - f_u since the fundamental has no drift term.

The first equation in (6) indicates that when *f*(*t*) lies inside the band, the central bank decides not to intervene so that ($dU = 0$ and $dL = 0$). Therefore, *f* is driven by pure Brownian motion. The second equation indicates that when *f*(*t*) hits f_u, the central bank uses *dU* in sales of foreign reserves to keep *f* within the band. Now suppose that at time 0, the initial state variable *f* is above the upper bound.

The central bank sells *dU* in foreign reserves to move *f* down to the upper boundary. Similarly, the third equation indicates that the central bank buys *dL* in foreign reserves in order to keep *f* at the lower boundary when *f* is below f_l. After time 0, the optimal policy requires infinitesimal *dU* and *dL* to keep the fundamental in the optimal band.

This characterization of the policy also requires the following technical conditions. The *dU* and *dL* control variables are nonnegative, right-continuous processes. *dU* and *dL* are composed of continuous parts ($dU = (U(t))' dt$) and discrete parts ($dU = \Delta U$). We will discuss later the description of the central bank's optimal policy.

The first step to find an optimal policy is to obtain the functional solution $V(f)$ from the Bellman equation and then to use the necessary optimality conditions to find the optimal control policy. Before proceeding, we will characterize a free-floating exchange rate regime as a simple case.

4. THE FREE-FLOATING EXCHANGE RATE REGIME

In a floating exchange rate regime, the central bank allows market forces to determine the exchange rate. Since there is no intervention available to the central bank under the free floating regime, the fundamental is driven by *v*(*t*). This section will examine how costly the flexible exchange rate regime is to the central bank. The loss function under the flexible exchange rate regime, *J*(*f*) is given by:

$$J(f:0,0) = E_0\int_0^{\infty} exp^{-rt}\,[(1-a)(e(t))^2 + a(i(t)-\bar{i})^2]\,dt \tag{7}$$[5]

To calculate the expected cost of maintaining the free-regime, the exchange rate solution *e*(*t*) has to be obtained. Since the law of motion governing the exchange rate equation is a linear function of *f*(*t*), the representation of equation (1) irrespective of the dynamics of *f* is

$$e(t) = 1/d\,E\int_t^{\infty} exp^{-\frac{(t-t)}{d}}\, f(t)\,dt \tag{8}$$

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[5] Note that $J(f{:}\ U, L) = J(f{:}\ 0, 0)$.

The saddle-path exchange rate solution to equation (8) is

$$e(t) = f(t) \tag{9}$$

The interest rate differential under a free-floating rate regime is obtained by comparing equation (9) and equation (1) with the open interest rate parity condition:

$$i(t) - i^{*} = 0 \tag{10}$$

Equation (10) implies that the central bank is able to eliminate the interest rate deviations if it sets $\overline{i}$ to be i^{*}. Substituting equation (10) into equation (7) yields the following loss function under the free-floating exchange rate regime:

$$J(f:0,0) = E_0 \int_0^{\infty} exp^{-rt}\,(1-a)e(t)^2\,dt \tag{11}$$

Before calculating the loss function, let us examine the dynamics of the free-floating exchange rate and the free-floating interest rate. We have already shown that the free-floating interest rate is constant. The dynamics of the exchange rate can be obtained by differentiating equation (9):

$$de(t) = s\ dw(t) \tag{12}$$

So the movements of the exchange rate are:

$$\begin{aligned} &E_0[e(t)] = e(0) \\ &Var_0[e(t)] = s^{2}t \end{aligned} \tag{13}$$

Equation (13) implies that under the free-floating exchange rate regime the (conditional) volatility of the exchange rate evaluated at time 0 tends to increase as time goes on. These wide and persistent fluctuations in currency values will ultimately have harmful effects on the foreign exchange market.

Next, the value of the loss function in the free-floating exchange rate system has to be calculated. To do so, we have to find the exchange rate deviation cost. The appendix shows how to calculate the value of the loss function under the free-floating exchange rate regime. According to this, the value of the loss function is

$$J(f:0,0) = (1-a)\left[\frac{f^2}{r} + \frac{s^2}{r^2}\right] \tag{14}$$

Equation (14) shows that the cost to the central bank of running a floating exchange regime will be determined by the central bank's discount factor *r*, the unpredictability of exchange rate movements per unit time s, and the degree of the central bank's concern about currency fluctuations, *1 - a*.

These results suggest that the larger the fluctuation of the home currency, and the more concerned about exchange fluctuation costs a central bank is, the larger the expected costs of a floating exchange regime.

5. OPTIMAL POLICY IN THE TARGET ZONE REGIME

In a target zone exchange rate regime, the central bank can, at any time, control *f*(*t*) through *dU* or *dL* (at a cost) to influence exchange rate movements. This implies that the fundamental fluctuates as a regulated Brownian motion. However any attempts to stabilize currency fluctuations under a target zone regime create deviations of short-term interest rates around a target rate. In order to minimize the expected discounted costs consisting of the two deviation costs and the intervention cost, the central bank has to choose an optimal intervention policy characterized by the optimal barriers of f_l and f_u. At any point in time it has the following options: (1) do nothing; (2) buy reserves; or (3) sell reserves.

A. Exchange Rate and Interest Rate Solution under the Target Zone Regime

Krugman (1987) showed that an exchange rate solution follows an S-shape inside the currency band. Let *e*(*f: U, L*) be the exchange rate solution when any arbitrary intervention point is predetermined. The use of the law of motion of the exchange rate subject to a regulated Brownian motion produces the following representation of *e*(*f: U, L*):

$$e(f:U,L) = 1/d\,E\int_0^{\infty} exp^{-\frac{t}{d}} f(t)\,dt \tag{15}$$

and it is subject to the following dynamics of the fundamental:

$$df(t) = s\ dw(t) - dU(t) + dL(t) \tag{16}$$

It follows that

$$\frac{d}{2} s^{2} e''(f) - e(f) + f = 0 \tag{17}$$

Equation (17) is the second order ordinary differential equation that contains the following exchange rate solution:[6]

$$e(f) = f + A\ \sinh(a\ f) \tag{18}$$

where the characteristic root of equation (18) is $a = \sqrt{2s^{2}/d}$. To pin down the value of A, we can invoke the tangency condition at the band that $e'(f_u) = 0$.

$$\begin{aligned} A &= -\frac{1}{a\ \cosh(a\ f_u)} \\ &= -\frac{\sec h\,(a\ f_u)}{a} < 0 \end{aligned} \tag{19}$$

The parameter $A \neq 0$ produces the ‘honeymoon’ effect, which generates the stabilization effect that the exchange rate is less responsive to the fundamental.

[6] Recall that sinh $(a\ f) = 1/2(exp^{a\,f} - exp^{-a\,f})$

Let us consider the derivation of the interest rate solution under a target zone regime. The domestic interest rate is defined as the foreign interest rate plus the expected rate of depreciation of the exchange rate if the uncovered interest parity holds with a risk-neutral situation. Using the interest parity condition and the target zone exchange rate solution, we obtain the target zone interest rate solution as follows:

$$\begin{aligned} i(f) &= \frac{Ede(f)}{dt} \\ &= i^{*} + \frac{e(f) - f}{d} \\ &= i^{*} + \frac{A \sinh(a\ f)}{d} \end{aligned} \tag{20}$$

So far we have explicitly obtained the solutions for interest rates and exchange rates under an arbitrary target zone. Using these solutions, we will be able to derive the exact form of the value function in the following section.

6. OPTION INVENTION POLICY

The central bank facing a trade-off between the benefits of stabilizing exchange rates and the cost of intervening will choose a band size supported by the optimal control policy to minimize the expected future discounted costs composed of the deviation and its control. More specifically, we will find conditions of the optimal band size under which the value function meets the optimality conditions.

The explicit form of the value function is given by:

$$\begin{aligned} V(f) &= \frac{k(f_u : A)}{r}\left(f^2 + \frac{s^2}{r}\right) + B\cosh(b\ f)\ \ for\ f \in [f_l, f_u] \\ &= \frac{k(f_u : A)}{r} + b(f - f_u)\ \ for\ f \geq f_u \\ &= \frac{k(f_u : A)}{r} - b(f + f_l)\ \ for\ f \leq f_l \end{aligned} \tag{21}$$

where $k(f_u\!: A)$ is given by:

$$k(f_u\!: A) = \left[(1-a)(1-\sec h\,(a\ f_u))^2 + a\left(i^* + \frac{\sec h\,(a\ f_u)}{d}\right)^2\right]$$

and the constant B is:

$$B = \frac{b - ek\frac{f_u}{r}}{b\ \sinh\,(b\ f_u)}$$

(See Appendix for a detailed derivation of the value function.) The necessary optimality condition that $V' = b$ yields:

$$2k(f_u\!: A)\frac{f_u}{r} + Bb\ \sinh\,(b\ f_u) = b \tag{22}$$

Given this value function, we can use the optimality condition and the smooth-pasting condition to derive the following optimal upper boundary:[7]

$$\frac{\tanh\,(b\ f_u)}{b} = f_u - \frac{br}{2k(f_u\!: A)} \tag{23}$$

Once the intervention band with $-f_u$ and f_u is chosen, we can use equations (18) and (19) to determine the size of the currency band characterized by e_l and e_u:

$$e_u - e_l = 2f_u - \frac{\tanh\,(a\ f_u) - \tanh\,(-a\ f_u)}{a} \tag{24}$$

[7] Note that $\tanh\,(b\ f_u) = \sinh\,(b\ f_u)/\cosh\,(b\ f_u)$

Figure 1 shows both the shape of the value function and the location of the optimal band. When the fundamental lies between f_l and f_u, it is optimal for the central bank to let it stray until it hits one of the edges of the band. Once the fundamental equals an upper or lower band and then an adverse shock occurs, the optimal policy requires the central bank to intervene infinitesimally to keep the fundamental from straying out of the band.

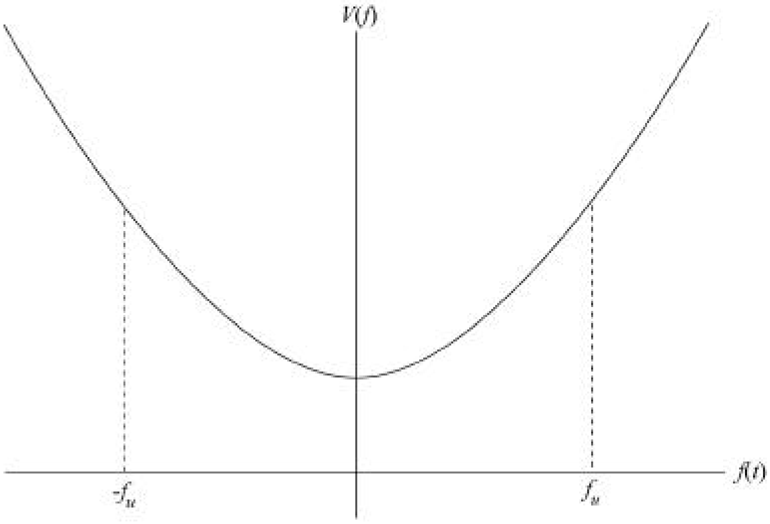

Figure 1. Value Function Under Target Zone Regime: V(f: U,L)

A. Implications

Miller and Zhang (1994) recently derived an analytical solution of the optimal band size and the associated value function. For purposes of comparison, let us rewrite their value function and the optimality condition on f_u according to our notation:

$$V(f: U,L) = (1 + Aa)\left(\frac{f^2}{r} + \frac{s^2}{r^2}\right) + B\cosh(b\ f) \tag{25}$$

Equation (25) represents the (minimal) cost function obtained from the assumption that the central bank seeks to minimize only the deviation of the exchange rate around a central parity. Miller and Zhang propose the following condition for the optimal barrier given the central bank's concern about the exchange rate deviation:

$$\frac{2(1+Aa)\,f_u}{r} + b\ B\sinh(b\ f_u) = b \tag{26}$$

Note that equations (25-26) are analogous to our corresponding results in equations (22-23). In their model the term Aa, which represents the 'honeymoon' effect, captures the stabilizing effect on the current exchange rate through the expectations about future intervention. Our model considers the destabilizing effect on the current interest rate through $(Aa/d)^2$ as well as the expectation effect on the exchange rate of future intervention. Their model is interpreted as a special case in the sense that the central bank is only concerned about the exchange rate deviation (In other words, the parameter $a = 0$ in their model).

Second, the model in this paper predicts that the optimal size of the band is wider than that of Miller and Zhang. Let us define f_u^0, f_u^* to be the optimal upper boundary in Miller and Zhang's and this model, respectively. The existence of the destabilizing effect on the interest rate, $(i^* + Aa/d)^2$, makes the marginal deviation cost in this model, V', greater than V' in their model for every f. This model thus leads to a delay in intervention, allowing a wider band. Therefore, this model proposes an optimal band width wider than that of Miller and Zhang.

7. WIDTH OF THE BAND: A NUMERICAL EXAMPLE

This section will employ the relation in equation (24) to discuss the response of f_u to changes in the structural parameters: s^2, r, d, b, and f. Unfortunately, it is impossible to obtain a closed form solution for the optimal upper boundary. This model uses a numerical approach to obtain a numerically approximate solution of f_u. Table 3 shows our numerical results for the width of the band and welfare costs for the given structural parameters.

A higher s^2 shifts the value function upward, but also increases f_u. This is illustrated in Table 3a. Suppose that the economy is at the optimal upper barrier. When an adverse shock to f occurs, the central bank intervenes to move it back to the upper boundary, while there is no intervention in the case of a favorable shock. This implies that with a higher variance, the expected change in the exchange rate would become increasingly negative. As a result, the expected marginal benefit arising from the intervention at the old upper barrier is less than the unit intervention cost. Hence, the central bank finds it optimal to delay its intervention. So, the band becomes wider in this case.

Table 3b shows the response of the optimal band to changes in the discount rate r with specific structural parameters. When r rises, the central bank discounts the expected future deviation costs more heavily, whereas a higher r does not affect the intervention costs since they are incurred immediately. So, the central bank finds it optimal to delay intervention. This implies that the optimal band is wider. Increases in b, shown in Table 3c, tend to widen the size of the band by raising the intervention cost.

A higher d decreases the cost associated with interest rate volatility, lowering the level of the value function. In addition, it reduces the expected marginal deviation costs relative to the unit intervention cost. In this situation, the central bank will find it optimal to delay its intervention. Table 3d supports this interpretation.

In Table 3e a higher value of a tends to increase the expected deviation costs by raising the central bank's perceived interest rate deviation cost. Since the change in a, alternatively, does not affect the intervention cost, the central bank will delay intervention. This causes the optimal band to be wider.

Table 3a. Response of Band and Welfare Costs to Changes in s^2

($a = 0.5$, $b = 1$, $d = 1$, $r = 0.1$)

s^2	Bandwidth (f_u)	Welfare ($V(f_u)$)
0.01	0.300	1.416
0.1	0.625	2.456
0.3	0.920	3.216

Table 3b. Response of Band and Welfare Costs to Changes in r

($a = 0.5$, $b = 1$, $d = 1$, $s^2 = 0.1$)

r	Bandwidth (f_u)	Welfare ($V(f_u)$)
0.01	0.597	22.754
0.1	0.625	2.456
0.2	0.680	1.406

Table 3c. Response of Band and Welfare Costs to Changes in *b*

($a = 0.5$, $r = 1$, $d = 1$, $s^2 = 0.1$)

b	Bandwidth (f_u)	Welfare (V(f_u))
0.01	0.120	0.465
1	0.625	2.456
2	0.795	3.547

Table 3d. Response of Band and Welfare Costs to Changes in *d*

($a = 0.5$, $b = 1$, $r = 0.1$, $s^2 = 0.1$)

d	Bandwidth (f_u)	Welfare (V(f_u))
0.01	0.559	2.795
1	0.625	2.456
3	0.747	1.240

Table 3e. Response of Band and Welfare Costs to Changes in *a*

($b = 1$, $r = 0.1$, $d = 1$, $s^2 = 0.1$)

a	Bandwidth (f_u)	Welfare (V(f_u))
0.1	0.546	3.379
0.5	0.625	2.456
0.8	0.865	1.600

8. SUMMARY AND CONCLUDING REMARKS

This paper has emphasized the tension between interest rate policy and exchange rate policy under a target zone exchange rate regime. The maintenance of the target zone exchange rate regime has two opposing effects on the financial markets. The positive effect on the market arises when a central bank stabilizes currency fluctuations. This stabilizing effect through the manipulation of market expectations is obtained at the expense of raising the volatility of the interest rate. So, the negative effect on the money market arises from the volatility of the domestic short-term interest rates due to exchange market intervention. Based on this intuition, this paper derived the optimal intervention strategy of a central bank with an infinite horizon facing proportional intervention costs. The optimal policy involves two boundaries for the fundamental characterized by two reflecting barriers. When the fundamental lies inside the band comprising a lower and upper barrier, the central bank finds it optimal not to intervene. When the fundamental hits the band, the central bank intervenes in order to keep it within the band.

A combination of factors determine the optimal size of the currency band: a variance parameter; the degree of speculative pressure characterized by the responsiveness of money demand with respect to the interest rate; the central bank's time discount rate; the central bank's concern about the uncertainty of the domestic interest rate induced by intervention; and the unit transaction cost in the foreign exchange market.

The optimal size of the intervention band depends positively on all of the following: the volatility of the fundamental; the discount rate; the degree of speculative pressure; and the importance of domestic money market conditions to the central bank.

APPENDIX

A. Derivation of the Law of Motion of the Exchange Rate

The law of motion of the exchange rate is composed of the following three basic equations:

$$m(t) - p(t) = ky(t) - d\,i(t) + e^*(t)$$

$$m^*(t) - p^*(t) = ky^*(t) - d\,i^*(t) + e^*(t)$$

$$e(t) = q(t) + p(t) - p^*(t)$$

$$\frac{E(de(t))}{dt} = (i(t) - i^*(t))$$

The first two equation describes the equilibrium condition for the domestic money market for each country, where $m(t)$ is the log of the domestic money supply, $p(t)$ the log of the domestic price level, $y(t)$ the log of the level of actual output, $i(t)$ the log of the domestic nominal interest rate, and $e(t)$ the disturbance of the domestic currency where x variables with x represents the foreign. The parameters k and d represent the sensitivity of the demand for money with respect to output and that of the money demand with respect to interest rate, respectively.

The third equation shows that the purchasing power parity condition holds, implying that the log of the nominal exchange rate, $e(t)$, always equals the log of the real exchange rate, $q(t)$, plus the log of the price differential between the domestic country and abroad, $p(t) – p^*$.

The fourth equation illustrates a currency arbitrage condition in which the expected rate of depreciation of the exchange rate, $E[de(t)]/dt$, equals the interest differential between the domestic country and abroad ($i(t) – i^*$) putting the result from the three equations into the right-hand side of the fourth equation, it follows that :

$$\frac{1}{d}\frac{Ede(t)}{dt} = m - m^*(t) + q(t) - e(t) + k(y - y^*) - (e - e^*)$$

$$e(t) = m - m^* - k(y - y^*) + (e - e^*) + q + \frac{1}{d}\frac{Ede(t)}{dt}$$

$$= m + (e - e^*) + q - m^*$$

Suppose that the shock to the economy $v(t) = (e – e^*) + q(e) - m^*(e)$ and $v(t)$ is assumed to be driven only by $e(t)$ term, whereas the rest of the terms are constant. Therefore

$$e(t) = f(t) + \frac{1}{d}\frac{Ede(t)}{dt}$$

where

$$f(t) = m(t) + v(t)$$

the log of the spot exchange rate at time t, $e(t)$, which reflects the structure of the economy, will be determined by both the fundamentals $f(t)$ and a speculative term proportional to the expected rate of depreciation in the exchange rate, $d\,E[de(t)] / dt$.

B. The Calculation of the Operating Costs of the Free-floating Regime

$$e(0) = 1/d\, E_0 \int_0^{\infty} (f(t)) e^{-\frac{rt}{d}} dt$$

The saddle-path exchange rate solution to the above equation is:

$$e(0) = f(0)$$

Let us consider the movements of the exchange rate. Since

$$E_0 e(t) = E_0 f(t)$$

it follows that

$$E_0 e(t) = e(0)$$

and

$$Var_0 (e(t)) = s^2 t$$

So the dynamics of the exchange rate solution is given by:

$$e(t) = e(0) + s \int_0^t dw(t)$$

and

$$e^2(t) = e^2(0) + 2e(0)s\int_0^t dw(t) + s^2\left(\int_0^t dw(t)\right)^2$$

$E(e^2(t))$ is given by:

$$E(e^2(t)) = e^2(0) + s^2 t$$

Therefore,

$$V(f:0,0) = (1-a)\int_0^\infty e^{-rt} e^2(0)dt$$

So, the value function under the free-floating regime is given by:

$$V(f:0,0) = (1-a)\left(\frac{e_0^2}{r} + \frac{s^2}{r^2}\right)$$

Expressing it in terms of $f(t)$ as follows:

$$V(f:0,0) = (1-a)\left(\frac{f_0^2}{r} + \frac{s^2}{r^2}\right)$$

C. Intuitive Derivation of Bellman Inequalities

We use a solution method for the optimal control of a Brownian motion process with linear control cost (Fleming et al. pp.317-318, 1993). It is shown that if $V(f: U, L)$ is convex and twice continuously differentiable, the value function satisfies the following Bellman inequalities:

$$0 = \min_{U,L} \left\{ \frac{s^{2}V''}{2} - rV + ai(f)^{2} + (1 - a)e(f)^{2} \right. \tag{27}$$
$$\left. + [- U(t)V' + U(t)b] + [L(t)V' + L(t)b] \right\}$$

which can be rearranged as follows:

$$0 = \min_{U,L} \left\{ \frac{s^{2}V''}{2} - rV + ai(f)^{2} + (1 - a)e(f)^{2} \right. \tag{28}$$
$$\left. + \min_{U \geq 0} U(t)[- V' + b] + \min_{L \geq 0} L(t)[V' + b] \right\}$$

Equation (28) implies either

$$\frac{s^{2}V''}{2} - rV - ai(f)^{2} + (1 - a)e(f)^{2} \geq 0 \tag{29}$$

or

$$U(t)\,[- V' + b] \geq 0 \tag{30}$$

or

$$L(t)\,[- V' + b] \geq 0 \tag{31}$$

CASE 1: The derivation of equation (29) can be explained on the following heuristic level: suppose that inside the band, f fluctuates during a small period dt and the associated value function is $V(f + df)$. Its discounted value function is $\exp^{-rdt} V(f + df)$. Since $V(f)$ is equal to or less than any loss function during the infinitesimal period, the principle of dynamic programming yields

$$V(f) \leq E\int_0^{dt} \exp^{-rdt}[ai(t)^2 + (1-a)e(t)^2]\,dt + E[\exp^{-rdt} V(f + df)] \tag{32}$$

Since under a no-control zone $dU = dL = 0$ it, follows that $df = s dw(t)$. Applying Ito’s lemma to $\exp^{-rdt} V(f + df)$ yields

$$d(\exp^{-rdt} V(f + df)) = \exp^{-rdt}[(\Gamma V - rV)dt + s\ V'\,dw] \tag{33}$$

where Γ is the differential generator of the Brownian motion f with:

$$\Gamma V = \frac{s^2 V''}{2} - rV + ai(f)^2 + (1-a)e(f)^2$$

Integrate equation (33) and take an expectation of the result:

$$E\int_0^{dt} d(\exp^{-rt} V(f + df))dt = E\int_0^{dt} \exp^{-rt}(\Gamma V - rV)dt \tag{34}$$

Rearrange it again:

$$E\int_0^{dt} d(\exp^{-rt} V(f + df))dt = E[\exp^{-rt} V(f + df)] - V(f) \tag{35}$$

Now, combining equation (33) and equation (34) yields

$$E[\exp^{-rt}\ V(f+df)] = +V(f) + E\int_0^{dt} \exp^{-rt}\,[(\Gamma V - rV)dt \qquad (36)$$

Substitution of equation (35) into equation (31) and letting dt go to zero yields:

$$\frac{s^2 V''}{2} - rV(f) - ai(f)^2 + (1-a)e(f)^2 \geq 0$$

CASE 2: Let us derive why equation (30) is the result in this situation. Suppose that f at time 0 exceeds the optimal upper barrier f_u by e. In this case, the central bank should buy that discrete amount of money needed to immediately boost $f - e$ to f_u at a cost of be and proceed optimally thereafter. The value function arising when the control policy is exerted should be equal to or less than the value function arising when no control is applied. In other words:

$$be + V(f - e) \leq V(f)$$

Applying Ito's lemma to $V(f-e)$ yields:

$$V(f) + (b - V'(f))\,e + o(e) \leq V(f)$$

This implies

$$(b - V'(f)) \leq 0 \qquad (37)$$

If the optimal policy is to intervene, the relation equation (37) must hold at an equality.

CASE 3: Similarly, we can consider the following case:

$$\min_{L \geq 0} L(t)[V' + b]$$

Based on the same logic as applied in Case 2, it is easy to demonstrate that

$$V' + b \geq 0 \quad \text{for} \quad f \leq f_l$$

Note that V' is negative for $f \leq f_l$.

Since at least one equality condition from Case 1-3 must hold, it follows that

$$\frac{s^2 V''(f)}{2} - rV + ai(f)^2 + (1-a)e(f)^2 = 0 \quad \text{if} \quad f \in [f_l, f_u] \tag{38}$$

$$V'(f) = b \quad \text{if} \quad f \geq f_u \tag{39}$$

$$-V'(f) = b \quad \text{if} \quad f \leq f_l \tag{40}$$

D. Necessary Conditions for and Optimum: The Bellman Inequalities

Derivation of Value Function

First of all, we have to find the general solution of $V(f)$ and then we can use the general solution and the optimality conditions to derive an exact form of the value function.

Let us use the Bellman equation to obtain the general solution of $V(f)$. It can be expressed as the sum of a homogenous solution, $V_h(f)$ and a particular solution, $V_p(f)$. $V_h(f)$ can be obtained by setting e^2 and i^2 in equation (38) to be zero. i.e.,

$$J(f:0,0) = (1-a)\left[\frac{f^2}{r} + \frac{s^2}{r^2}\right] \tag{41}$$

We can obtain the homogenous solution as follows:[8]

$$\begin{aligned} V_h(f) &= C(\exp^{a\,f(t)} - \exp^{a\,f(t)}) \\ &= B\cosh(b\,f(t)) \end{aligned} \tag{42}$$

where B is the constant which will be determined from the boundary condition and a, b are the roots of the following characteristic equation.

$$1/2s^2z^2 - r = 0$$

There are two roots for the equation: We can obtain the particular solution V_p by suspecting that it is of the form:

$$V_p = c_1 f^2 + c_2 \tag{43}$$

It must follow that V_p satisfies the following relation:

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[8] Let us guess that the homogenous solution is as follows:

$$V_h(f) = C\exp^{a\,f(t)} + D\exp^{b\,f(t)}$$

Plugging the candidate homogenous function into equation (38), we know that $D = -C$. In addition , the heperbolic cosine function of $f(t)$ is given by :

$$\cosh(b\,f) = 1/2(\exp^{b\,f(t)} + \exp^{-b\,f(t)})$$

For notational convenience, let $A = C/2$. Then we obtain equation (38).

$$0 = - rV_p(f) + \frac{s^2 V_p''(f)}{2} + (1 - a)e(f)^2 + ai(f)^2 \tag{44}$$

To solve for the particular solution, we must first of all express the exchange rate and the interest rate solution in terms of the fundamental. For small target zone regimes, it is possible to obtain the following linear target zone exchange rate solution by taking a Taylor expansion of sinh($a\ f$):

$$\sinh(a\ f(t)) = a\ f(t) \tag{45}$$

when $f(t)$ is around 0.[9] The combination of equations (40-41) and the exchange and interest rate equations (18) and (21) in section 5 yields the following particular solution:

$$V_p = k(f_u : A)\left(\frac{f^2}{r} + \frac{s^2}{r^2}\right) \tag{46}$$

where

$$k(f_u : A) = \left[(1 - a)(1 + Aa)^2 + a\left(i^* + \frac{Aa}{d}\right)^2\right] \tag{47}$$

where A is given by equation (19). So, the general solution of the loss function under the target zone exchange regime is given by:

$$V(f : U, L) = k\left(\frac{f(t)^2}{r} + \frac{s^2}{r^2}\right) + B\cosh(a\ f) \tag{48}$$

---

[9] This approach is also used by Svesson (1989) and Miller and Zhang (1994).

Based on the above result, we can obtain the following solution, including the optimal control policy and the associated value function:

$$\begin{aligned} V(f: U, L) &= k\left(\frac{f(t)^2}{r} + \frac{s^2}{r^2}\right) + B\cosh(a\ f) \text{ for } f \in [f_l, f_u] \\ &= \bar{V}(f_u) + b(f - f_u) \text{ for } f \geq f_u \\ &= \bar{V}(- f_u) - b(f + f_u) \text{ for } f \leq - f_u \end{aligned}$$

We can then use the value-matching condition that $V(f_u) = \bar{V}(f_u)$ to obtain $\bar{V}(f_u)$:

$$\bar{V}(f_u) = \frac{k(f_u: A) f_u^2}{r} \tag{49}$$

These four equations are just enough to determine the four unknown variables, the two constants B and $-B$ and two intervention points f_u and f_l. So the explicit form of the value function is given by

$$\begin{aligned} V(f) &= \frac{k(f_u: A)}{r}\left(f^2 + \frac{s^2}{r}\right) + B\cosh(b\ f) \text{ for } f \in [f_l, f_u] \\ &= \frac{k(f_u: A)}{r} + b(f - f_u) \text{ for } f \geq f_u \qquad (50) \\ &= \frac{k(f_u: A)}{r} - b(f + f_u) \text{ for } f \leq f_u \end{aligned}$$

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