

EXCHANGE RATE PASS-THROUGH IN AN INTERNATIONAL DUOPOLY MODEL WITH BRAND LOYALTY

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In many markets, consumers who have previously purchased from one firm have (or perceive) costs of switching to a competitor's product. This study explicitly analyzes, in an international duopoly model with brand loyalty, the effect of rival exchange rate on exchange rate pass-through. In the case of the imperfect foresight, the exchange rate pass-through is affected by the exchange rate uncertainty. Due to the brand loyalty, current price decisions will affect future profits through market shares. The expected future profit is affected by expected competition situations that depend on the interactive movement of future exchange rates. [F31, F12]

1. INTRODUCTION

Since the advent of floating exchange rates, firms based in different countries have faced notably large fluctuations in currency values. Exchange rate changes are usually perceived as cost shocks for a foreign firm producing in its home country and selling in its export market. When the exchange rate changes, the firm may choose to pass the cost shock into its selling prices; it is called exchange rate pass-through (PT). In general, it is widely observed that import prices in importer's currency move very little compared to movements in exchange rates (incomplete pass-through).

A number of authors have studied the underlying relationship between exchange rates and prices of internationally traded goods trying to explain the imperfect pass-through. The theoretical explanations of incomplete pass-through have emphasized the role of market structure and product differentiation. The major contributions are Dornbusch (1987), Krugman (1987,1989), Baldwin (1988), Baldwin and Krugman (1989), Dixit (1989a, 1989b), Fischer (1989), and Froot and Klemperer (1989).

Although there has been much research on exchange rate pass-through, no literature paying attention to rival's exchange rate was found. The existing exchange rate pass-through (PT) literature is based on an imperfect competition

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model of a foreign firm and a domestic firm, including only a bilateral exchange rate. However, if other foreign firms exist in the market, the foreign firm's pricing behavior will be affected by rival country exchange rates. In the international export market, an exporter may often face other foreign firms rather than domestic firms as major rivals. The substitutability between export goods may be much higher than the one between an export good and a domestic good. Then, rival exchange rate may give a significant effect to the firm's pricing decision through strategic interaction. The importance of strategic interaction in an export market was mentioned by Goldberg and Knetter (1997, p. 1265).

International economists typically impose the Armington assumption - i.e., they assume that products within an industry are differentiated according to the country of production. An extreme interpretation of the Armington assumption implies that goods produced in different countries represent different markets. ... The competition from these other sources is accounted for by including the prices of substitutes in the set of demand shifters or rotators; but this treatment fails to capture the strategic interaction.

This study explicitly analyzes the effect of rival exchange rate on exchange rate PT. The basic framework of this study is adapted from Froot and Klemperer (1989)'s brand loyalty model, however, two bilateral exchange rates are considered in this model, introducing the uncertainties. First, section 2 shows the effects of rival exchange rates due to strategic interaction. When the rival's exchange rate moves in the same (opposite) direction with its own change, the PT is magnified (minimized). If the change of rival's exchange rate is in the opposite direction to the change of the firm's exchange rate and relatively large enough, the exchange rate pass-through is perverse even with elastic demand. Therefore, considering rival exchange rates explains that exchange rate PT can be both of greater than unity and perverse. In Froot and Klemperer (1989) and section 2 of this paper, an unrealistic assumption, perfect foresight, is made. However, in section 3, I drop this assumption and instead assume that the firm's pricing decision must be made before future exchange rates (but after current exchange rates) are known. Interestingly, with imperfect foresight we can see that PT is affected by the covariance and variances of both its own and rival's exchange rate. In section 4 some concluding remarks are drawn.

2. PERFECT FORESIGHT MODEL AND THE EFFECT OF RIVAL EXCHANGE RATE

This study considers a two-period duopoly model with brand loyalty.¹ In many markets, consumers who have previously purchased from one firm have (or

¹Klemperer (1995) explained and illustrated the different types of switching costs, or reasons for "brand loyalty", that consumers may face.

perceive) costs of switching to a competitor's product, even when the two firm's products are functionally identical. Examples of switching costs include the learning cost incurred by switching to a new make of computer after having learned to use one make, and the transactions cost of closing an account with one bank and opening another with a competitor. Another kind of switching cost arises when uncertainty about product quality makes consumers reluctant to switch to untested products. These brand loyalties give firms a degree of market power over their repeat-purchasers, and mean that firms' current market shares are important determinants of their future profits. This model is a variant of Froot and Klemperer (1989). However, in this model, two different bilateral exchange rates are involved because each firm is based in a different foreign country.

There are two firms: a country j firm and a country k firm, which compete with differentiated products in the export (say, U.S.) market.² Own currency marginal costs of producer j and k are \mathbf{g}^j and \mathbf{g}^k , respectively.³ Consumers are assumed to exhibit a certain brand loyalty. Demand and thus profits in the second period depend on first-period sales. Firms behave noncooperatively and act simultaneously.

The export firms' problems will be:

$$\text{Max } \Pi^j = (e_{j1}p_{j1} - \mathbf{g}^j)q_{j1}(p_{j1}, p_{k1}) + I^j \left[(e_{j2}p_{j2} - \mathbf{g}^j)q_{j2}(S^j, p_{j2}, p_{k2}) \right], \quad (1a)$$

$$\text{Max } \Pi^k = (e_{k1}p_{k1} - \mathbf{g}^k)q_{k1}(p_{j1}, p_{k1}) + I^k \left[(e_{k2}p_{k2} - \mathbf{g}^k)q_{k2}(S^k, p_{j2}, p_{k2}) \right], \quad (1b)$$

where I^i = discount factor of firm i , e_{it} = t -period exogenous exchange rates defined in units of country i currency per dollar, p_{it} = price of good i in period t , q_{it} = quantity demanded for good i in period t , S^i = market share of firm i in period 1.

The subgame of period two is solved first. In period two, the second-period dollar prices are chosen to maximize the own currency second-period profits:

$$\text{Max}_{p_{j2}} \Pi_2^j = (e_{j2}p_{j2} - \mathbf{g}^j)q_{j2}(S^j(p_{j1}, p_{k1}), p_{j2}, p_{k2}), \quad (2a)$$

²Although the existence of domestic (U.S.) firms does not change the main economic points that the PT is affected by rival's exchange rate, we implicitly assume that the substitutability between export goods is much higher than the one between an export good and a domestic good.

³The foreign producers' home market is separated on the technological side and may thus be neglected.

$$\text{Max}_{p_{k2}} \Pi_2^k = (e_{k2} p_{k2} - \mathbf{g}^k) q_{k2} (S^k(p_{j1}, p_{k1}), p_{j2}, p_{k2}). \quad (2b)$$

Solving the associated first order conditions for the problems in equation (2), the second-period equilibrium prices are derived as functions of first-period prices, second-period exchange rates and second-period own currency marginal costs. The reduced form solutions are the followings:

$$p_{j2}^* = p_{j2}(p_{j1}, p_{k1}, e_{j2}, e_{k2}, \mathbf{g}^j, \mathbf{g}^k), \quad (3a)$$

$$p_{k2}^* = p_{k2}(p_{j1}, p_{k1}, e_{j2}, e_{k2}, \mathbf{g}^j, \mathbf{g}^k). \quad (3b)$$

In the first period, firms maximize the present discounted value of own-currency profits by choosing the first-period prices:

$$\text{Max}_{p_{j1}} \Pi^j = (e_{j1} p_{j1} - \mathbf{g}^j) q_{j1}(p_{j1}, p_{k1}) + I^j \Pi_2^j [S^j(p_{j1}, p_{k1}), p_{j2}^*, p_{k2}^*, e_{j2}], \quad (4a)$$

$$\text{Max}_{p_{k1}} \Pi^k = (e_{k1} p_{k1} - \mathbf{g}^k) q_{k1}(p_{j1}, p_{k1}) + I^k \Pi_2^k [S^k(p_{j1}, p_{k1}), p_{j2}^*, p_{k2}^*, e_{k2}], \quad (4b)$$

where Π_2^i = second- period own currency profits of firm i .

The first order condition will be:

$$\Pi_j^j = e_{j1} \left(\frac{\partial q_{j1}}{\partial p_{j1}} p_{j1} + q_{j1} \right) - \mathbf{g}^j \frac{\partial q_{j1}}{\partial p_{j1}} + I^j \frac{\partial \Pi_2^j}{\partial p_{j1}} = 0, \quad (5a)$$

$$\Pi_k^k = e_{k1} \left(\frac{\partial q_{k1}}{\partial p_{k1}} p_{k1} + q_{k1} \right) - \mathbf{g}^k \frac{\partial q_{k1}}{\partial p_{k1}} + I^k \frac{\partial \Pi_2^k}{\partial p_{k1}} = 0. \quad (5b)$$

It is assumed that $\partial \Pi_2^i / \partial p_{il} = [(\partial \Pi_2^i / \partial S^i)(\partial S^i / \partial p_{il}) + (\partial \Pi_2^i / \partial p_{s2}^*)(\partial p_{s2}^* / \partial p_{il})] < 0$ holds; or the first negative term dominate the second positive term. Therefore, firms choose lower prices than they would if market share had no value. It is further assumed that the second order conditions for profit maximization are fulfilled, and that the actions of the duopolists are strategic complements, i.e. the marginal profit of an own price increase rises with a higher price of the rival: $\partial^2 \Pi^i / \partial p_{il} \partial p_{sl} \equiv \Pi_{is}^i > 0$, $i, s = j, k$, $i \neq s$.

To calculate the comparative static reactions for the changes of exchange rates, we differentiate equation (5) and get:

$$dp_{jl} = \frac{1}{\Delta} \left\{ \Pi_{kk}^k q_{jl} (\mathbf{e}_l^j - 1) de_{jl} - \left[\Pi_{kk}^k \mathbf{I}^j \frac{\partial^2 \Pi_2^j}{\partial p_{jl} \partial e_{j2}} - \Pi_{jk}^j \mathbf{I}^k \frac{\partial^2 \Pi_2^k}{\partial p_{kl} \partial e_{j2}} \right] de_{j2} \right. \\ \left. + \Pi_{jk}^j q_{kl} (1 - \mathbf{e}_l^k) de_{kl} + \left[\Pi_{jk}^j \mathbf{I}^k \frac{\partial^2 \Pi_2^k}{\partial p_{kl} \partial e_{k2}} - \Pi_{kk}^k \mathbf{I}^j \frac{\partial^2 \Pi_2^j}{\partial p_{jl} \partial e_{k2}} \right] de_{k2} \right\}, \quad (6a)$$

$$dp_{kl} = \frac{1}{\Delta} \left\{ \Pi_{jj}^j q_{kl} (\mathbf{e}_l^k - 1) de_{kl} - \left[\Pi_{jj}^j \mathbf{I}^k \frac{\partial^2 \Pi_2^k}{\partial p_{kl} \partial e_{k2}} - \Pi_{kj}^k \mathbf{I}^j \frac{\partial^2 \Pi_2^j}{\partial p_{jl} \partial e_{k2}} \right] de_{k2} \right. \\ \left. + \Pi_{kj}^k q_{jl} (1 - \mathbf{e}_l^j) de_{jl} + \left[\Pi_{kj}^k \mathbf{I}^j \frac{\partial^2 \Pi_2^j}{\partial p_{jl} \partial e_{j2}} - \Pi_{jj}^j \mathbf{I}^k \frac{\partial^2 \Pi_2^k}{\partial p_{kl} \partial e_{j2}} \right] de_{j2} \right\}. \quad (6b)$$

where $\Delta = \Pi_{jj}^j \Pi_{kk}^k - \Pi_{jk}^j \Pi_{kj}^k > 0$ by a stability condition, and \mathbf{e}_l^i = the elasticity of first-period import demand for good i .

The third and fourth terms of equation (6) reflect the effect of rival's exchange rate change. If these terms are zero, the expression will be similar to Froot and Klemperer's (1989). However, if the change of exchange rate comes from the U.S. policy (e.g., an increase in the money supply in the U.S.), $Cov(e_j, e_k)$ will be generally positive, and the third and fourth terms will be significant. Also, when the change has its origin in rival country's domestic policy, rival's exchange rate may change without any change of its own currency value relative to dollar. This model suggests that the price may be changed by rival's exchange rate even when there is no change of its own exchange rate. A major and indeed important factor for exchange rate PT has simply been missed. If rival's (third country) exchange rate is not considered, theoretical analyses are misconceived, and empirical studies

are biased and inefficient.⁴ Particularly, empirical tests based on a bilateral trade and an exchange rate may face serious mis-specification problems if rival exchange rate is not hold as a constant.

For a temporary shock ($de_{j2} = de_{k2} = 0$ or $Cov(e_{j1}, e_{j2}) = Cov(e_{j1}, e_{k2}) = 0$) the comparative static is simplified as:

$$\frac{dp_{j1}}{de_{j1}} = \frac{1}{\Delta} \left\{ \prod_{kk}^k q_{j1}(\mathbf{e}_I^j - I) + \prod_{jk}^j q_{kl}(I - \mathbf{e}_I^k) \frac{de_{kl}}{de_{j1}} \right\}, \quad (7a)$$

$$\frac{dp_{kl}}{de_{j1}} = \frac{1}{\Delta} \left\{ \prod_{jj}^j q_{kl}(\mathbf{e}_I^k - I) \frac{de_{kl}}{de_{j1}} + \prod_{kj}^k q_{j1}(I - \mathbf{e}_I^j) \right\}, \quad (7b)$$

If $de_{kl} / de_{j1} > 0$ (or $Cov[e_{j1}, e_{kl}] > 0$) and $\mathbf{e}_I^j, \mathbf{e}_I^k > 1$ (< 1), the exchange rate pass-through is more normal (perverse) than the case that we do not consider the change of rival's exchange rate. This also implies that pass-through of greater than unity is possible in some case. More interesting is the case of $de_{kl} / de_{j1} < 0$ (or $Cov[e_{j1}, e_{kl}] < 0$). The elasticity of first-period import demand is no longer a necessary and sufficient condition for the "normal" PT of a temporary exchange rate shock. For example, consider a case that e_{j1} increases by 10% and e_{kl} decreases by 20%. $\mathbf{e}_I^j > 1$ does not guarantee the decrease of p_{j1} due to the second term of (7a). While an increase of e_{j1} may induce the firm to decrease p_{j1} , the increase of p_{kl} motivated by the decrease of e_{kl} may result in an increase in p_{j1} . Also, prices of the firm j and k do not always move in the same direction. If the relative change of e_{kl} is large enough, firm j may have perverse pass-through while firm k has normal exchange rate pass-through.⁵ Even though the goods are strongly substitutable, we can observe, in some level, that firm j decreases the price while firm k increases the price of a good. Of course, this is related to an empirical issue. If the rival exchange rate is hold constant as in an empirical test, perverse pass-through in terms of exporters is not possible in the elastic demand area. Again, it emphasizes that rival exchange rate should be considered in the pass-through estimation. These results are similar for permanent shock.

⁴In some empirical studies, the effect of rival exchange rate is partially reflected by rival prices (usually domestic price). However, there exists simultaneous bias problem because it fails to capture the strategic interaction.

⁵If duopoly (or oligopoly) firms produce in an elastic area (better match with the duopoly theory), Froot and Klemperer (1989) can not explain the perverse exchange rate pass-through.

3. IMPERFECT FORESIGHT MODEL AND THE EFFECT OF EXCHANGE RATE UNCERTAINTY

In Froot and Klemperer (1989) and section 2 of this paper, it is assumed that the level of the exchange rate at successive instants of time are known (or perfectly forecasted). While theories need simplifying assumptions, this is unrealistic. Therefore, in this section, I will assume that a firm's pricing decision is made after current exchange rates are known, but before future exchange rates are known. In the two period model, the events are as follows: $e_{j1}, e_{k1} \rightarrow p_{j1}, p_{k1} \rightarrow e_{j2}, e_{k2} \rightarrow p_{j2}, p_{k2}$; when each firm chooses p_{j1} and p_{k1} respectively, they know e_{j1} and e_{k1} but do not know e_{j2} and e_{k2} .⁶

The export firms' problems (1) will be modified as in (8):

$$\begin{aligned} \text{Max}_{p_{j1}, p_{j2}} E[\Pi^j] = & E_{e_{j2}, e_{k2}} \left\{ (e_{j1} p_{j1} - \mathbf{g}^j) q_{j1}(p_{j1}, p_{k1}) \right. \\ & \left. + \mathbf{I}^j \left[(e_{j2} p_{j2} - \mathbf{g}^j) q_{j2}(S^j, p_{j2}, p_{k2}) \right] \right\}, \end{aligned} \quad (8a)$$

$$\begin{aligned} \text{Max}_{p_{k1}, p_{k2}} E[\Pi^k] = & E_{e_{j2}, e_{k2}} \left\{ (e_{k1} p_{k1} - \mathbf{g}^k) q_{k1}(p_{j1}, p_{k1}) \right. \\ & \left. + \mathbf{I}^k \left[(e_{k2} p_{k2} - \mathbf{g}^k) q_{k2}(S^k, p_{j2}, p_{k2}) \right] \right\}. \end{aligned} \quad (8b)$$

In the period two, second-period dollar prices are chosen to maximize the own currency second-period profits. The first order conditions are:

$$\frac{\partial \Pi_2^j}{\partial p_{j2}} = e_{j2} q_{j2} + (e_{j2} p_{j2} - \mathbf{g}^j) \frac{\partial q_{j2}}{\partial p_{j2}} = 0 \quad (9a)$$

$$\frac{\partial \Pi_2^k}{\partial p_{k2}} = e_{k2} q_{k2} + (e_{k2} p_{k2} - \mathbf{g}^k) \frac{\partial q_{k2}}{\partial p_{k2}} = 0 \quad (9b)$$

⁶Some careful readers may think that the problem of order and payment lags does not justify this structure. Although this paper does not deal with hedging issues, the existence of currency futures market strongly supports this game structure. Indeed, whereas the short term futures market is easily available and practical, long term futures market is not available or very costly.

In the first period, firms maximize the present discounted value of own-currency profits by choosing first-period prices. Using the envelope theorem we find the first-order necessary conditions:

$$\begin{aligned} \frac{\partial E[\Pi^j]}{\partial p_{j1}} &= e_{j1}q_{j1} + (e_{j1}p_{j1} - \mathbf{g}^j) \frac{\partial q_{j1}}{\partial p_{j1}} \\ &+ \mathbf{I}^j E_{e_{j2}, e_{k2}} \left[(e_{j2}p_{j2}^* - \mathbf{g}^j) \left(\frac{\partial q_{j2}}{\partial S^j} \frac{\partial S^j}{\partial p_{j1}} + \frac{\partial q_{j2}}{\partial p_{k2}^*} \frac{\partial p_{k2}^*}{\partial p_{j1}} \right) \right] = 0, \end{aligned} \quad (10a)$$

$$\begin{aligned} \frac{\partial E[\Pi^j]}{\partial p_{k1}} &= e_{k1}q_{k1} + (e_{k1}p_{k1} - \mathbf{g}^k) \frac{\partial q_{k1}}{\partial p_{k1}} \\ &+ \mathbf{I}^k E_{e_{j2}, e_{k2}} \left[(e_{k2}p_{k2}^* - \mathbf{g}^k) \left(\frac{\partial q_{k2}}{\partial S^k} \frac{\partial S^k}{\partial p_{k1}} + \frac{\partial q_{k2}}{\partial p_{j2}^*} \frac{\partial p_{j2}^*}{\partial p_{k1}} \right) \right] = 0. \end{aligned} \quad (10b)$$

Finally we can get the reduced form solution of first-period prices as follows:⁷

$$\begin{aligned} p_{j1}^* &= p_{j1} (e_{j1}, e_{k1}, E[e_{j2}], E[e_{k2}], \text{Var}[e_{j2}], \text{Var}[e_{k2}], \\ &\text{Cov}[e_{j2}, e_{k2}], \mathbf{g}^j, \mathbf{g}^k, \mathbf{I}^j, \mathbf{I}^k), \end{aligned} \quad (11a)$$

$$\begin{aligned} p_{k1}^* &= p_{k1} (e_{j1}, e_{k1}, E[e_{j2}], E[e_{k2}], \text{Var}[e_{j2}], \text{Var}[e_{k2}], \\ &\text{Cov}[e_{j2}, e_{k2}], \mathbf{g}^j, \mathbf{g}^k, \mathbf{I}^j, \mathbf{I}^k). \end{aligned} \quad (11b)$$

In the case of imperfect foresight, second-period exchange rates are not known, and each firm's first-period price decision relies upon the expected exchange rates,

⁷ p_{j1}^* and p_{k1}^* depend on the distribution of e_{j2} and e_{k2} . Assuming that it can be captured by the five parameters ($E[e_{j2}]$, $E[e_{k2}]$, $\text{Var}[e_{j2}]$, $\text{Var}[e_{k2}]$, $\text{Cov}[e_{j2}, e_{k2}]$), we can get equation (11).

and variances and covariance of exchange rates. Due to brand loyalty, a current period pricing decision affects not only current profit but also future profit through its impact on a market share. At that time, the expected future profit (or the value of current market share) is effected by expected competition situations which depend on the interactive movement of future exchange rates.

To obtain more precise results, I analyze exchange rate PT using an example of Tivig (1996) in which demand functions are linear and symmetric in current prices. This demand structure corresponds to the Hotelling model of differentiated products presented as the “Linear City Case” in Tirole (1990, chapter 7.1.1):

$$q_{i1} = \frac{1}{2} \cdot p_{i1} + p_{s1} \Rightarrow Q_1 = q_{j1} + q_{k1} = 1, S^i = \frac{q_{i1}}{Q_1} = q_{i1},$$

$$q_{i2} = S^i \cdot p_{i2} + p_{s2} \Rightarrow Q_2 = q_{j2} + q_{k2} = 1; i, s = j, k; i \neq s,$$

where Q_1 and Q_2 are the first- and second-period market demand, respectively, normalized at one. For the first step, we obtain the solution of the second-period prices:

$$\begin{bmatrix} p_{j2}^* \\ p_{k2}^* \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} S^j + \frac{\mathbf{g}^j}{e_{j2}} \\ S^k + \frac{\mathbf{g}^k}{e_{k2}} \end{bmatrix}. \quad (12)$$

Then, using equation (12) we can get the solution of the first-period prices:

$$\begin{bmatrix} p_{j1}^* \\ p_{k1}^* \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 2 - \frac{2}{9} \mathbf{b}_k & 1 - \frac{2}{9} \mathbf{b}_j \\ 1 - \frac{2}{9} \mathbf{b}_k & 2 - \frac{2}{9} \mathbf{b}_j \end{bmatrix} \begin{bmatrix} \frac{3-2\mathbf{b}_j}{6} + C_{j1} + \frac{2}{9} \mathbf{b}_j C_{j2} - \frac{2}{9} \mathbf{b}_j C_{k2} \mathbf{q}_k \\ \frac{3-2\mathbf{b}_k}{6} + C_{k1} + \frac{2}{9} \mathbf{b}_k C_{k2} - \frac{2}{9} \mathbf{b}_k C_{j2} \mathbf{q}_j \end{bmatrix}, \quad (13)$$

where $\Delta \equiv 3 - (2/9)(\mathbf{b}_j + \mathbf{b}_k)$, $C_{it} \equiv \mathbf{g}^i/E[e_{it}]$: marginal cost in \$, $\mathbf{b}_i \equiv \mathbf{I}_i(E[e_{i2}]/e_{i1})$, and $\mathbf{q}_i \equiv E[e_{s2}/e_{i2}](E[e_{i2}]/E[e_{s2}])$.

Equation (13) shows that the change of exchange rates affects each firm's pricing decision through [1] the change of dollar costs (C_{it}), [2] the effect on real interest rates (\mathbf{b}_i), and [3] the uncertainty effect (\mathbf{q}_i which depends on the distribution of exchange rate). If costs adjust instantaneously, C_{it} is independent of exchange rates, and the cost effect due to the change of the exchange rate is zero. If costs adjust only to the anticipated change of exchange rate, C_{i1} is changed by the unanticipated movement in e_{i1} , while C_{i2} is not affected; that is, a permanent shock where cost adjusts with a one period lag is comparable to a transitory shock. Equation (13) also shows that prices decrease as uncertainties increase. In this study, the particular attention is paid to the uncertainty effect on exchange rate pass-through. Assuming $\mathbf{I}^j = \mathbf{I}^k = 1$ to simplify, we can get the solution of first-period prices. Then, equation (13) can be modified as equation (13)':

$$\begin{bmatrix} p_{j1}^* \\ p_{k1}^* \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{2}{9}\bar{e}_{k2} - 2e_{k1} & \frac{2}{9}\bar{e}_{j2} - e_{j1} \\ \frac{2}{9}\bar{e}_{k2} - e_{k1} & \frac{2}{9}\bar{e}_{j2} - 2e_{j1} \end{bmatrix} \begin{bmatrix} -\frac{1}{2}e_{j1} + \frac{1}{3}\bar{e}_{j2} - \frac{11}{9}\mathbf{g}^j + \frac{2}{9}\mathbf{g}^k E\left[\frac{e_{j2}}{e_{k2}}\right] \\ -\frac{1}{2}e_{k1} + \frac{1}{3}\bar{e}_{k2} - \frac{11}{9}\mathbf{g}^k + \frac{2}{9}\mathbf{g}^j E\left[\frac{e_{k2}}{e_{j2}}\right] \end{bmatrix}, \quad (13)'$$

where $\Delta = ((2/9)\bar{e}_{j2} - 2e_{j1})((2/9)\bar{e}_{k2} - 2e_{k1}) - ((2/9)\bar{e}_{j2} - e_{j1})((2/9)\bar{e}_{k2} - e_{k1})$ and $\bar{e}_{i2} = E[e_{i2}]$.

The $E[e_{i2}/e_{s2}]$ term depends on the distribution of exchange rate uncertainties. For the further analysis, here I assume the random shocks of exchange rates follow a bivariate normal distribution. Then, $E[e_{i2}/e_{s2}] \approx E[e_{i2}](1/E[e_{s2}]) + (Var[e_{s2}]/E[e_{s2}]^3) - (Cov[e_{i2}, e_{s2}]/E[e_{s2}]^2)$ by Taylor extension.⁸ From (13)' we can see that

⁸First, assuming normal distribution, by Taylor extension we can get:

$$(1) \quad Cov\left[e_i, \frac{1}{e_s}\right] = E\left\{(e_i - E[e_i])\left(\frac{1}{e_s} - E\left[\frac{1}{e_s}\right]\right)\right\} \approx -\frac{Cov[e_i, e_s]}{E[e_s]^2} \quad \text{and} \quad 2) \quad E\left[\frac{1}{e_s}\right] \approx \frac{1}{E[e_s]} + \frac{Var[e_s]}{E[e_s]^2}$$

$$\text{Using (1) and (2) we can get:} \quad E\left[\frac{e_i}{e_s}\right] = E[e_i] E\left[\frac{1}{e_s}\right] + Cov\left[e_i, \frac{1}{e_s}\right] \approx E[e_i] \left(\frac{1}{E[e_s]} + \frac{Var[e_s]}{E[e_s]^2}\right) - \frac{Cov[e_i, e_s]}{E[e_s]^2}.$$

$$\begin{aligned} \text{Proof of (1):} \quad (e_i - E[e_i])\left(\frac{1}{e_s} - E\left[\frac{1}{e_s}\right]\right) &\approx (e_i - E[e_i])\left(\frac{1}{E[e_s]} - E\left[\frac{1}{e_s}\right]\right) + (e_i - E[e_i])\left(-\frac{1}{E[e_s]^2}\right)(e_s - E[e_s]) \\ &\quad + \frac{1}{2}(e_i - E[e_i])\left(\frac{-2}{E[e_s]^3}\right)(e_s - E[e_s])^2 \quad \text{Then,} \end{aligned}$$

the exchange rate PT will be affected by the covariance and variances of exchange rates as well as current own- and rival bilateral exchange rates and expected future exchange rates. To illustrate, I show how the covariance and variance of exchange rates affect the exchange rate PT.

PROPOSITION 1. *For a temporary exchange rate shock, [A] If normal pass-through prevails (or $e_i^i > 1$), around a symmetric equilibrium, (1) exchange rate PT increases as the covariance of exchange rates increases; (2) exchange rate PT decreases as the variance of rival's exchange rate increases; and (3) exchange rate PT increases as the variance of own exchange rate increases. [B] If perverse pass-through prevails (or $e_i^i < 1$), around a symmetric equilibrium, (1) perverse exchange rate PT decreases as the covariance of exchange rates increases; (2) perverse exchange rate PT increases as the variance of rival's exchange rate increases; and (3) the direction of exchange rate PT is ambiguous as the variance of own exchange rate increases.*

PROOF: In this "Linear City case", equation (10) is written as

$$\frac{\partial E[\Pi^j]}{\partial p_{jl}} = e_{jl} \left(\frac{1}{2} - 2p_{jl} + p_{kl} \right) + \mathbf{g}^j + \mathbf{I}^j \underset{e_{j2}, e_{k2}}{E} \left[\left(e_{j2} p_{j2}^* - \mathbf{g}^j \right) \left(-\frac{2}{3} \right) \right], \quad (10a)'$$

$$\frac{\partial E[\Pi^k]}{\partial p_{kl}} = e_{kl} \left(\frac{1}{2} - 2p_{kl} + p_{jl} \right) + \mathbf{g}^k + \mathbf{I}^k \underset{e_{j2}, e_{k2}}{E} \left[\left(e_{k2} p_{k2}^* - \mathbf{g}^k \right) \left(-\frac{2}{3} \right) \right]. \quad (10b)'$$

To calculate the comparative static reactions to a temporary change in the exchange rate, we differentiate (10)' and get: Assuming $de_{kl} = 0$,

$$\begin{aligned} E \left\{ (e_i - E[e_i]) \left(\frac{1}{e_s} - E \left[\frac{1}{e_s} \right] \right) \right\} &\approx - \frac{\text{Cov}[e_i, e_s]}{E[e_s]^2} + E \left\{ (e_i - E[e_i]) (e_s - E[e_s])^2 \right\} = - \frac{\text{Cov}[e_i, e_s]}{E[e_s]^2} \text{ because} \\ E \left\{ (e_i - E[e_i]) (e_s - E[e_s])^2 \right\} &= E \left\{ E \left[(e_i - E[e_i]) (e_s - E[e_s])^2 \mid e_s \right] \right\} = E \left\{ E \left[(e_i - E[e_i]) \mid e_s \right] (e_s - E[e_s])^2 \right\} \\ &= E \left\{ \frac{\mathbf{s}_{e_i e_s}}{\mathbf{s}_{e_s e_s}} (e_s - E[e_s]) (e_s - E[e_s])^2 \right\} = E \left\{ \frac{\mathbf{s}_{e_i e_s}}{\mathbf{s}_{e_s e_s}} (e_s - E[e_s])^3 \right\} = 0 \end{aligned}$$

$$\begin{bmatrix} dp_{jl} \\ dp_{kl} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{2}{9}\bar{e}_{k2} - 2e_{kl} & \frac{2}{9}\bar{e}_{j2} - e_{jl} \\ \frac{2}{9}\bar{e}_{k2} - e_{kl} & \frac{2}{9}\bar{e}_{j2} - 2e_{jl} \end{bmatrix} \begin{bmatrix} \left(-\frac{1}{2} + 2p_{jl} - p_{kl}\right) de_{jl} \\ 0 \end{bmatrix}.$$

$$\begin{aligned} \text{Then, } PT^j &= \frac{dp_{jl}}{de_{jl}} \cdot \frac{e_{jl}}{p_{jl}} = \frac{e_{jl}}{\Delta \cdot p_{jl}} \left(\frac{2}{9}\bar{e}_{k2} - 2e_{kl} \right) (\mathbf{e}_l^j - 1) q_{jl} \\ &= \frac{e_{jl}}{\Delta} \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \left(\frac{(1 + 2p_{kl})}{2p_{jl}} - 2 \right) \begin{matrix} \leq 0 \\ > 0 \end{matrix} \text{ as } \mathbf{e}_l^j \begin{matrix} \geq 1 \\ < 1 \end{matrix}. \end{aligned}$$

$$\begin{aligned} (1) \frac{\partial PT^j}{\partial \text{Cov}[e_{j2}, e_{k2}]} &= \frac{e_{jl}}{\Delta} \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{2p'_{kl} \cdot 2p_{jl} - (1 + 2p_{kl}) \cdot 2p'_{jl}}{(2p_{jl})^2} \\ &= \frac{e_{jl}}{\Delta} \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{4p'_{kl}p_{jl} - 4p_{kl}p'_{jl} - 2p'_{jl}}{(2p_{jl})^2} < 0 \text{ around symmetry.} \end{aligned}$$

$$\begin{aligned} \text{Where } p'_{jl} &= \frac{\partial p_{jl}}{\partial \text{Cov}[e_{j2}, e_{k2}]} = \frac{e_{jl}}{\Delta \cdot 9} \left\{ \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{\mathbf{g}^k}{(\bar{e}_{k2})^2} + \left(e_{jl} - \frac{2}{9}\bar{e}_{j2} \right) \cdot \frac{\mathbf{g}^j}{(\bar{e}_{j2})^2} \right\} > 0, \\ p'_{kl} &= \frac{\partial p_{kl}}{\partial \text{Cov}[e_{j2}, e_{k2}]} = \frac{2}{\Delta \cdot 9} \left\{ \left(e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{\mathbf{g}^k}{(\bar{e}_{k2})^2} + \left(2e_{jl} - \frac{2}{9}\bar{e}_{j2} \right) \cdot \frac{\mathbf{g}^j}{(\bar{e}_{j2})^2} \right\} > 0. \end{aligned}$$

For symmetric firms, $p_{jl} \approx p_{kl}$ and $p'_{jl} \approx p'_{kl}$. Then,

$$\frac{\partial PT^j}{\partial \text{Cov}[e_{j2}, e_{k2}]} \approx -\frac{e_{jl}}{\Delta} \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{2p'_{jl}}{(2p_{jl})^2} < 0.$$

$$(2) \frac{\partial PT^j}{\partial \text{Var}[e_{k2}]} = \frac{e_{jl}}{\Delta} \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{2p''_{kl} \cdot 2p_{jl} - (1+2p_{kl}) \cdot 2p''_{jl}}{(2p_{jl})^2} > 0 \text{ around symmetry.}$$

$$\text{Where } p''_{jl} = \frac{\partial p_{jl}}{\partial \text{Var}[e_{k2}]} = \frac{1}{\Delta} \left(\frac{2}{9}\bar{e}_{k2} - 2e_{kl} \right) \frac{2}{9} \mathbf{g}^k \frac{\bar{e}_{j2}}{(\bar{e}_{k2})^3} < 0,$$

$$p''_{kl} = \frac{\partial p_{kl}}{\partial \text{Var}[e_{k2}]} = \frac{1}{\Delta} \left(\frac{2}{9}\bar{e}_{k2} - e_{kl} \right) \frac{2}{9} \mathbf{g}^k \frac{\bar{e}_{j2}}{(\bar{e}_{k2})^3} < 0.$$

Thus, $\text{sign} \frac{\partial PT^j}{\partial \text{Var}[e_{k2}]} = \text{sign} [p''_{kl} \cdot 2p_{jl} - p''_{jl}(1+2p_{kl})] > 0$ because $|p''_{kl}| < |p''_{jl}|$ and $2p_{jl} < (1+2p_{kl})$ around symmetry.

$$(3) \frac{\partial PT^j}{\partial \text{Var}[e_{j2}]} = \frac{e_{jl}}{\Delta} \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{2p'''_{kl} \cdot 2p_{jl} - (1+2p_{kl}) \cdot 2p'''_{jl}}{(2p_{jl})^2} ? 0 \text{ around symmetry.}$$

$$\text{Where } p'''_{jl} = \frac{\partial p_{jl}}{\partial \text{Var}[e_{j2}]} = \frac{1}{\Delta} \left(\frac{2}{9}\bar{e}_{j2} - e_{jl} \right) \frac{2}{9} \mathbf{g}^j \frac{\bar{e}_{k2}}{(\bar{e}_{j2})^3} < 0,$$

$$p'''_{kl} = \frac{\partial p_{kl}}{\partial \text{Var}[e_{j2}]} = \frac{1}{\Delta} \left(\frac{2}{9}\bar{e}_{j2} - 2e_{jl} \right) \frac{2}{9} \mathbf{g}^j \frac{\bar{e}_{k2}}{(\bar{e}_{j2})^3} < 0.$$

Then, $\text{sign} \frac{\partial PT^j}{\partial \text{Var}[e_{j2}]} = \text{sign} [p'''_{kl} \cdot 2p_{jl} - p'''_{jl}(1+2p_{kl})]$: we can not determine because

$$|p'''_{kl}| > |p'''_{jl}| \text{ and } 2p_{jl} < (1+2p_{kl}). \frac{\partial PT^j}{\partial \text{Var}[e_{j2}]} \begin{matrix} \geq \\ < \end{matrix} 0 \text{ as } \frac{p'''_{kl} < (1+2p_{kl})}{p'''_{jl} > 2p_{jl}}.$$

However, if $e_j^j \geq 1$, $\frac{\partial PT^j}{\partial \text{Var}[e_{j2}]} < 0$.

If $e_j^j \geq 1 \Leftrightarrow -\frac{1}{2} + 2p_{j1} - p_{k1} \geq 0 \Leftrightarrow p_{j1} \geq \frac{1}{2}$ around symmetry.

$$\begin{aligned} \text{Then, sign} \left[p_{kl}''' \cdot 2p_{j1} - p_{j1}''' (1 + 2p_{kl}) \right] &= \text{sign} \left[2p_{j1} \left(\frac{2}{9} \bar{e}_{j2} - 2e_{j1} \right) - (1 + 2p_{kl}) \left(\frac{2}{9} \bar{e}_{j2} - 2e_{j1} \right) \right] \\ &= \text{sign} \left[-2p_{j1}e_{j1} - \frac{2}{9} \bar{e}_{j2} + e_{j1} \right] < 0. \quad \text{Q.E.D.} \end{aligned}$$

PROPOSITION 2. *For a permanent exchange rate shock, (1) exchange rate PT increases as the covariance of exchange rates increases; (2) exchange rate PT increases as the variance of rival's exchange rate increases; and (3) exchange rate PT decreases as the variance of own exchange rate increases.*

PROOF: For a permanent change of one exchange rate, a perverse exchange rate PT in this model is not possible.⁹ To calculate the comparative static reactions to a permanent change in the exchange rate, we differentiate (10)' and get: Assuming $de_k = 0$ and $de_{j1} = d\bar{e}_{j2}$,

$$\begin{bmatrix} dp_{j1} \\ dp_{kl} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{2}{9} \bar{e}_{k2} - 2e_{k1} & \frac{2}{9} \bar{e}_{j2} - e_{j1} \\ \frac{2}{9} \bar{e}_{k2} - e_{k1} & \frac{2}{9} \bar{e}_{j2} - 2e_{j1} \end{bmatrix} \begin{bmatrix} \left(-\frac{1}{6} + \frac{16}{9} p_{j1} - \frac{7}{9} p_{kl} + \frac{2}{9} g^k \left(\frac{1}{\bar{e}_{k2}} + \frac{\text{Var}(e_{k2})}{\bar{e}_{k2}^3} \right) \right) de_j \\ \left(\frac{2}{9} g^j \left(-\frac{\bar{e}_{k2}}{\bar{e}_{j2}^2} - \frac{3\bar{e}_{k2} \text{Var}(e_{j2})}{\bar{e}_{j2}^4} + \frac{2\text{Cov}(e_{j2}, e_{k2})}{\bar{e}_{j2}^3} \right) \right) de_j \end{bmatrix}$$

$$\text{Then, } PT^j = \frac{dp_{j1}}{de_j} \cdot \frac{e_j}{p_{j1}} = \frac{e_j}{\Delta} \left[\left(2e_{k1} - \frac{2}{9} \bar{e}_{k2} \right) \frac{\left(\frac{1}{6} + \frac{7}{9} p_{kl} - \frac{2}{9} g^k \left(\frac{1}{\bar{e}_{k2}} + \frac{\text{Var}(e_{k2})}{\bar{e}_{k2}^3} \right) \right)}{p_{j1}} - \frac{16}{9} \right]$$

⁹See Froot and Klemperer (1989).

$$(1) \frac{\partial PT^j}{\partial \text{Cov}[e_{j2}, e_{k2}]} = \frac{e_j}{\Delta p_{jl}^2} \left\{ \left[\left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{7}{9} p'_{kl} - \left(e_{jl} - \frac{2}{9}\bar{e}_{j2} \right) \frac{4}{9} \frac{\mathbf{g}^j}{\bar{e}_{j2}^3} \right] p_{jl} \right. \\ \left. - \left[\left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \left(\frac{1}{6} + \frac{7}{9} p_{kl} - \frac{2}{9} \mathbf{g}^k \left(\frac{1}{\bar{e}_{k2}} + \frac{\text{Var}(e_{k2})}{\bar{e}_{k2}^3} \right) \right) \right] \right. \\ \left. + \left(e_{jl} - \frac{2}{9}\bar{e}_{j2} \right) \frac{2}{9} \mathbf{g}^j \left(\frac{\bar{e}_{k2}}{\bar{e}_{j2}^2} + \frac{3\bar{e}_{k2}\text{Var}(e_{j2})}{\bar{e}_{j2}^4} - \frac{2\text{Cov}(e_{j2}, e_{k2})}{\bar{e}_{j2}^3} \right) \right] p'_{jl} \right\}$$

$$\text{Where } p'_{jl} = \frac{\partial p_{jl}}{\partial \text{Cov}[e_{j2}, e_{k2}]} = \frac{2}{\Delta \cdot 9} \left\{ \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{\mathbf{g}^k}{(\bar{e}_{k2})^2} + \left(e_{jl} - \frac{2}{9}\bar{e}_{j2} \right) \cdot \frac{\mathbf{g}^j}{(\bar{e}_{j2})^2} \right\} > 0,$$

$$p'_{kl} = \frac{\partial p_{kl}}{\partial \text{Cov}[e_{j2}, e_{k2}]} = \frac{2}{\Delta \cdot 9} \left\{ \left(e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{\mathbf{g}^k}{(\bar{e}_{k2})^2} + \left(2e_{jl} - \frac{2}{9}\bar{e}_{j2} \right) \frac{\mathbf{g}^j}{(\bar{e}_{j2})^2} \right\} > 0.$$

For symmetric firms, $p_{jl} \approx p_{kl}$, $p'_{jl} \approx p'_{kl}$, $\mathbf{g}^j = \mathbf{g}^k$ and $e_{jl} = e_{kl} = \bar{e}_{j2} = \bar{e}_{k2}$ initially. Then, the first [] term is negative. Next, we use the fact that (a) $\text{Var}(e_{i2}) < \bar{e}_{i2}^2$ from the assumption that e_i is a positive number and follow a normal distribution, and (b) non-negative profit condition. Then, second [] term is positive. Therefore, we can get $\partial PT^j / \partial \text{Cov}[e_{j2}, e_{k2}] < 0$.

$$(2) \frac{\partial PT^j}{\partial \text{Var}[e_{k2}]} = \frac{e_j}{\Delta p_{jl}^2} \left\{ \left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \frac{2}{9} \frac{\mathbf{g}^k}{\bar{e}_{k2}^3} \left[\frac{1}{\Delta} \left(\frac{25}{9} e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \bar{e}_{j2} - 1 \right] \right. \\ \left. - \left[\left(2e_{kl} - \frac{2}{9}\bar{e}_{k2} \right) \left(\frac{1}{6} - \frac{16}{9} p_{jl} + \frac{7}{9} p_{kl} - \frac{2}{9} \mathbf{g}^k \left(\frac{1}{\bar{e}_{k2}} + \frac{\text{Var}(e_{k2})}{\bar{e}_{k2}^3} \right) \right) \right] \right. \\ \left. + \left(e_{jl} - \frac{2}{9}\bar{e}_{j2} \right) \frac{2}{9} \mathbf{g}^j \left(\frac{\bar{e}_{k2}}{\bar{e}_{j2}^2} + \frac{3\bar{e}_{k2}\text{Var}(e_{j2})}{\bar{e}_{j2}^4} - \frac{2\text{Cov}(e_{j2}, e_{k2})}{\bar{e}_{j2}^3} \right) \right] p'_{jl} \right\}$$

$$\text{where } p''_{j1} = \frac{\partial p_{j1}}{\partial \text{Var}[e_{k2}]} = \frac{1}{\Delta} \left(\frac{2}{9} \bar{e}_{k2} - 2e_{k1} \right) \frac{2}{9} \mathbf{g}^k \frac{\bar{e}_{j2}}{(\bar{e}_{k2})^3} < 0,$$

$$p''_{k1} = \frac{\partial p_{k1}}{\partial \text{Var}[e_{k2}]} = \frac{1}{\Delta} \left(\frac{2}{9} \bar{e}_{k2} - e_{k1} \right) \frac{2}{9} \mathbf{g}^k \frac{\bar{e}_{j2}}{(\bar{e}_{k2})^3} < 0.$$

With the fact that initially $e_{j1} = e_{k1} = \bar{e}_{j2} = \bar{e}_{k2}$, the first [] term is zero. Next, second [] term is negative because $dp_{j1}/de_j < 0$. Then, we can get $\partial PT^j / \partial \text{Var}[e_{k2}] < 0$.

$$(3) \frac{\partial PT^j}{\partial \text{Var}[e_{j2}]} = \frac{e_j}{\Delta p_{j1}^2} \left\{ \left[\left(2e_{k1} - \frac{2}{9} \bar{e}_{k2} \right) \frac{7}{9} p'''_{k1} + \left(e_{j1} - \frac{2}{9} \bar{e}_{j2} \right) \frac{2}{9} \mathbf{g}^j \frac{3\bar{e}_{k2}}{\bar{e}_{j2}^4} \right] p_{j1} \right. \\ \left. - \left[\left(2e_{k1} - \frac{2}{9} \bar{e}_{k2} \right) \left(\frac{1}{6} + \frac{7}{9} p_{k1} - \frac{2}{9} \mathbf{g}^k \left(\frac{1}{\bar{e}_{k2}} + \frac{\text{Var}(e_{k2})}{\bar{e}_{k2}^3} \right) \right) \right. \right. \\ \left. \left. + \left(e_{j1} - \frac{2}{9} \bar{e}_{j2} \right) \frac{2}{9} \mathbf{g}^j \left(\frac{\bar{e}_{k2}}{\bar{e}_{j2}^2} + \frac{3\bar{e}_{k2} \text{Var}(e_{j2})}{\bar{e}_{j2}^4} - \frac{2\text{Cov}(e_{j2}, e_{k2})}{\bar{e}_{j2}^3} \right) \right] p''_{j1} \right\}$$

$$\text{where } p'''_{j1} = \frac{\partial p_{j1}}{\partial \text{Var}[e_{j2}]} = \frac{1}{\Delta} \left(\frac{2}{9} \bar{e}_{j2} - e_{j1} \right) \frac{2}{9} \mathbf{g}^j \frac{\bar{e}_{k2}}{(\bar{e}_{j2})^3} < 0,$$

$$p'''_{k1} = \frac{\partial p_{k1}}{\partial \text{Var}[e_{j2}]} = \frac{1}{\Delta} \left(\frac{2}{9} \bar{e}_{j2} - 2e_{j1} \right) \frac{2}{9} \mathbf{g}^j \frac{\bar{e}_{k2}}{(\bar{e}_{j2})^3} < 0.$$

Similarly with (1), we can $\partial PT^j / \partial \text{Var}[e_{j2}] > 0$.

Q.E.D.

These main results are due to brand loyalty and imperfect foresight for exchange rates. The decision of current price influences future profit through market shares as well as a current profit. The effects of exchange rate uncertainty on pass-through depend on the curvature of future profitability on future exchange rates. For example, for a temporary shock, the second period profit is a convex function of own exchange rate. With the switching costs, each

firm may act as a monopolist in its first-period share of the market, and a monopolist's profit is a convex function of its cost. Therefore, greater uncertainty of own exchange rate increases the value of market shares, hence lowers a current price to increase the market share. Likewise, for a temporary shock, the second period profit is a concave function of a rival exchange rate and PT decreases as rival's exchange rate uncertainty increases. A risk neutral profit maximizer increases the PT and attacks the market when covariance between own- and rival exchange rate is high. Intuitively, the firm will not hesitate to change its price because a higher covariance guarantee a more stable or insured competition condition in the future market. For example, *ceteris paribus*, if a Korean firm competes against a Japanese rather than a German firm, the pass-through is higher assuming that the won/dollar rate is more closely correlated to the yen/dollar rate.

4. CONCLUDING REMARKS

If other foreign firms exist in an imperfect competition market, the foreign firm's pricing behavior is affected by rival countries' exchange rates through strategic interactions. This study explicitly analyzed the effect of rival exchange rate both in the case of perfect foresight and in the case of imperfect foresight. Although the assumption of perfect foresight is not realistic, it makes the analysis easier and tractable. In section 2, my results add to those of Froot and Klemperer's (1989). First, pass-through of greater than unity is possible in some cases. It is shown that if the change of own exchange rate is in the same direction with rival's, the normal exchange rate pass-through is magnified by the rival, and the PT can be greater than one. Indeed, some empirical studies have shown that the PTs are over 100 % for some industries.¹⁰ Second, if the change of own exchange rate is in the opposite direction to that of the rival's and is relatively small compared to the rival's, perverse exchange rate pass-through happens even with elastic demands. Also, it is possible that the firm chooses a perverse pass-through strategy even with a permanent exchange rate change. If duopoly (or oligopoly) firms produce in the elastic area (which is a better match with the duopoly theory), Froot and Klemperer (1989) and Tivig (1996) can not explain the perverse exchange rate pass-through.

In the case of the imperfect foresight, the exchange rate PT is affected by the covariance and variances of exchange rates as well as current exchange rates and expected future exchange rates. Due to brand loyalty, current price decisions will affect future profits through market shares. The expected future profit is effected by expected competition situations which depend on the interactive movement of future exchange rates. *Proposition 1* and *2* summarize how the interactive movement of exchange rates affects the exchange rate PT.

¹⁰See Marquez (1991), Feenstra (1989) and Phillips (1988).

Most importantly, this research emphasizes the importance of market structure in exchange rate pass-through studies. The specification of market structure changes the analytical form and also raises questions concerning the definition of PT. Most existing theoretical and empirical studies on exchange rate PT do not pay an attention to the competition between goods that produced in different source countries.

Much additional research needs to be done in this area. In particular, the extension to an n-period dynamic game would be useful, and a general equilibrium approach and cost structure with the consideration for stochastic functional form of exchange rate would be more in keeping with the environment in which real world firms must make decisions.

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